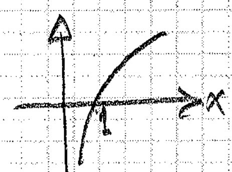
L'HOPITAL'S RULE (PART I)

$$\lim_{x\to a} \frac{\overset{\to 0}{f(x)}}{\underset{\to 0}{g(x)}} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

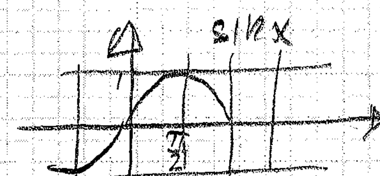
$$\lim_{x\to a} \frac{\overset{\to \pm\infty}{f(x)}}{\underset{\to \pm\infty}{g(x)}} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

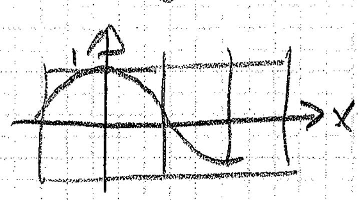
Examples: calculate

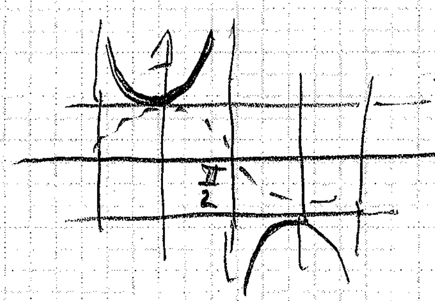
1) $\lim\limits_{x\to 1} \frac{\overset{\to 0}{\ln x}}{\underset{\to 0}{1-x}} \overset{L'H}{=} \lim\limits_{x\to 1} \frac{\frac{1}{x}}{-1} = \frac{1}{(-1)} \lim\limits_{x\to 1} \frac{1}{x} = -1$

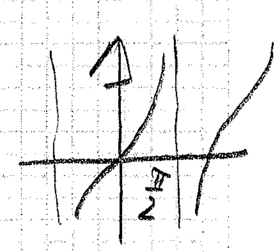


2) $\lim\limits_{x\to 0} \frac{\overset{\to 0}{\sin x}}{\underset{\to 0}{x}} \overset{L'H}{=} \lim\limits_{x\to 0} \cos x = 1$









3) $\lim\limits_{x\to \frac{\pi}{2}} \frac{\sec x}{1+\tan x}$

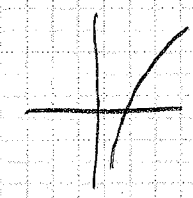
$$\lim_{x\to \frac{\pi}{2}^-} \frac{\overset{\to +\infty}{\sec x}}{\underset{\to \infty}{1+\tan x}} \overset{L'H}{=} \lim_{x\to \frac{\pi}{2}^-} \frac{\cancel{\sec x}\cdot \tan x}{\sec^{\cancel{2}} x} = \lim_{x\to \frac{\pi}{2}^-} \frac{\tan x}{\sec x}$$

$$= \lim_{x \to \frac{\pi}{2}^-} \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \lim_{x \to \frac{\pi}{2}^-} \frac{\sin x}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{1} = \lim_{x \to \frac{\pi}{2}} \sin x = 1$$

$$\lim_{x \to \frac{\pi}{2}^+} \frac{\sec x}{1 + \tan x} \overset{LH}{=} \lim_{x \to \frac{\pi}{2}^+} \sin x = 1$$

INDETERMINATE FORMS OF TYPE $0 \cdot \infty$

4) $L = \lim_{x \to 0^+} (\underbrace{x}_{\to 0} \cdot \underbrace{\ln x}_{\to -\infty})$



$$\boxed{\text{TRICK: } a \cdot b = \frac{b}{\frac{1}{a}} = \frac{a}{\frac{1}{b}}}$$

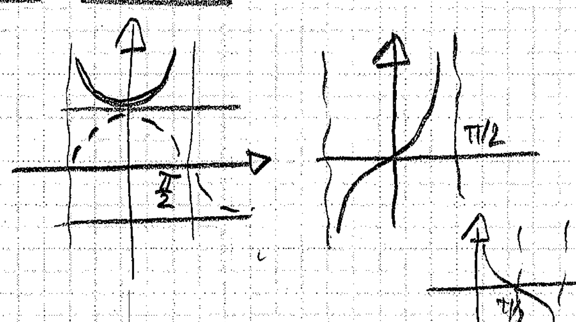
$$\left(\frac{1}{x}\right)' = (x^{-1})' = -1\, x^{-2} = -\frac{1}{x^2}$$

$$L = \lim_{x \to 0^+} \left(\frac{\ln x \to -\infty}{\frac{1}{x} \to +\infty} \right) \overset{LH}{=} \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = -\lim_{x \to 0^+} \left[\frac{1}{x} \cdot \frac{x^2}{1} \right]$$

$$= -\lim_{x \to 0^+} x = 0$$

INDETERMINATE FORMS OF TYPE $\infty - \infty$

5) $L = \lim_{x \to (\frac{\pi}{2})^-} (\underbrace{\sec x}_{\to +\infty} - \underbrace{\tan x}_{\to +\infty})$



$$= \lim_{x \to (\frac{\pi}{2})^-} \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x} \right) =$$

$$= \lim_{x \to (\frac{\pi}{2})^-} \frac{1 - \sin x \to 0}{\cos x \to 0} \overset{LH}{=} \lim_{x \to (\frac{\pi}{2})^-} \frac{-\cos x}{-\sin x} = \lim_{x \to \frac{\pi}{2}} \cot x = 0$$