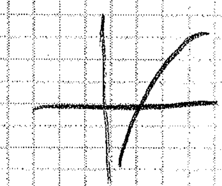
L'HOPITAL'S RULE (PART II)

$\frac{0}{0}$ $\frac{\infty}{\infty}$ $0 \cdot \infty$ $\infty - \infty$ PART I

0^0 ∞^0 1^∞ PART II

1) $\lim_{x \to 0^+} x^x$



Form 0^0

$a \cdot b = \frac{b}{\frac{1}{a}}$

$y = x^x \Rightarrow \ln y = \ln x^x = x \ln x$

$\lim_{x \to 0^+} \ln y = \lim_{x \to 0^+} [x \ln x] = \lim_{x \to 0^+} \left[\frac{\ln x}{\frac{1}{x}} \right] \overset{L'H}{=}$

$0 \cdot \infty$ $\frac{\infty}{\infty}$

$= \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = -\lim_{x \to 0^+} \left[\frac{1}{x} \frac{x^2}{1} \right]$

$\left(\frac{1}{x}\right)' = (x^{-1})' = (-1)x^{-2}$

$\log_2 8 = 3 \Rightarrow$
$\Rightarrow 8 = 2^3$

$= -\lim_{x \to 0^+} x = 0 \Rightarrow \lim_{x \to 0^+} \ln y = 0$

$\ln y = 0 \Rightarrow$
$\Rightarrow y = e^0 = 1$

$\Rightarrow \lim_{x \to 0^+} y = e^0 = 1$

2) $\lim\limits_{x \to +\infty} (\ln x)^{1/x}$

$$y = (\ln x)^{1/x} \Rightarrow \ln y = \ln (\ln x)^{1/x} = \frac{1}{x} \cdot \ln(\ln x)$$

$$\lim_{x \to +\infty} \ln y = \lim_{x \to +\infty} \left[\frac{1}{x} \cdot \ln(\ln x) \right] = \lim_{x \to +\infty} \frac{\ln(\ln x)}{x}$$

$0 \cdot \infty$ $\qquad$ $\frac{\infty}{\infty}$

$$\overset{L'H}{=} \lim_{x \to +\infty} \frac{\frac{1}{\ln x} \cdot \frac{1}{x}}{1} = \lim_{x \to \infty} \left[\frac{1}{\ln x} \cdot \frac{1}{x} \right] = 0 \cdot 0 = 0$$

$$\lim_{x \to +\infty} \ln y = 0 \Rightarrow \lim_{x \to +\infty} y = e^0 = 1$$

3) $\lim\limits_{x \to +\infty} \left(1 - \frac{2}{x}\right)^x$

1^∞