LOGARITHMIC DIFFERENTIATION

Ex: $y = x^x$

Find $\frac{dy}{dx}$

$\cancel{(x^n)' = n x^{n-1}}$ $n =$ constant

$\cancel{(b^x)' = b^x \cdot \ln b}$ $b =$ constant

$\ln y = \ln(x^x)$ $\qquad \log_b x^n = n \log_b x$

$\ln y = x \cdot \ln x$

Implicit Diff

$\frac{d}{dx}(\ln y) = \frac{d}{dx}(x \cdot \ln x)$

Product Rule

$\frac{d}{dx}(\ln x) = \frac{1}{x}$ Chain R.

but $\frac{d}{dx}(\ln y) = \frac{1}{y} \cdot \frac{dy}{dx}$

$(f \cdot g)' = f' g + f \cdot g'$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{dx}{dx} \cdot \ln x + x \frac{d}{dx}(\ln x)$

$\frac{1}{y} \frac{dy}{dx} = \ln x + x \cdot \frac{1}{x} = \ln x + 1 = 1 + \ln x$

$$\frac{dy}{dx} = (1 + \ln x) \cdot y = (1 + \ln x)\, x^x$$

Ex: $y = \frac{x^3 (3-x^2)^3}{(2x+3)^2 (4-2x)^5}$ $\qquad y' = \frac{dy}{dx}$ $\qquad (f \cdot g)' = f' g - f \cdot g'$

$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$

$$y' = \frac{[x^3 (3-x^2)^3]' [(2x+3)^2 (4-2x)^5] - x^3 (3-x^2)^3 [(2x+3)^2 (4-2x)^5]'}{[(2x+3)^2 (4-2x)^5]^2}$$

$$\ln y = \ln\left(\frac{x^3(3-x^2)^3}{(2x+3)^2(4-2x)^5}\right)$$

Properties of Logs

$\log_b(x\cdot y) = \log_b x + \log_b y$

$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

$\log_b x^n = n\log_b x$

$$\ln y = \ln\left(x^3\cdot(3-x^2)^3\right) - \ln\left[(2x+3)^2(4-2x)^5\right]$$

$$\ln y = \ln x^3 + \ln(3-x^2)^3 - \left[\ln(2x+3)^2 + \ln(4-2x)^5\right]$$

$$\ln y = 3\ln x + 3\ln(3-x^2) - 2\ln(2x+3) - 5\ln(4-2x)$$

Do implicit Diff

$$\frac{1}{y}\cdot\frac{dy}{dx} = 3\cdot\frac{1}{x} + 3\,\frac{1}{3-x^2}(-2x) - 2\,\frac{1}{2x+3}\cdot 2 - 5\cdot\frac{1}{4-2x}(-2)$$

$$\frac{dy}{dx} = \left(\frac{3}{x} - \frac{6}{3-x^2} - \frac{4}{2x+3} + \frac{10}{4-2x}\right)\cdot\frac{x^3(3-x^2)^3}{(2x+3)^2(4-2x)^5}$$