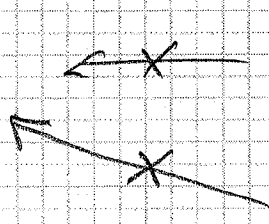


LOGARITHMIC DIFFERENTIATION

Ex: $y = x^x$

Find $\frac{dy}{dx}$



$(x^n)' = n x^{n-1}$
 $n = \text{constant}$

$(b^x)' = b^x \cdot \ln b$
 $b = \text{constant}$

$\ln y = \ln(x^x)$

$\log_b x^n = n \log_b x$

$\ln y = x \cdot \ln x$

Implicit diff

$\frac{d}{dx}(\ln y) = \frac{d}{dx}(x \cdot \ln x)$
 ↑
 Product Rule

$\frac{d}{dx}(\ln x) = \frac{1}{x}$ Chain R.
 ↓

but $\frac{d}{dx}(\ln y) = \frac{1}{y} \cdot \frac{dy}{dx}$

$(f \cdot g)' = f'g + f \cdot g'$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{dx}{dx} \cdot \ln x + x \cdot \frac{d}{dx}(\ln x)$

$\frac{1}{y} \cdot \frac{dy}{dx} = \ln x + x \cdot \frac{1}{x} = \ln x + 1 = 1 + \ln x$

$\frac{dy}{dx} = (1 + \ln x) \cdot y = (1 + \ln x) x^x$

Ex: $y = \frac{x^3 (3-x^2)^3}{(2x+3)^2 (4-2x)^5}$

$y' = \frac{dy}{dx} \quad (f \cdot g)' = f'g + f \cdot g'$

$(\frac{f}{g})' = \frac{f'g - f \cdot g'}{g^2}$

$y' = \frac{[x^3 (3-x^2)^3]' [(2x+3)^2 (4-2x)^5] - x^3 (3-x^2)^3 [(2x+3)^2 (4-2x)^5]'}{[(2x+3)^2 (4-2x)^5]^2}$

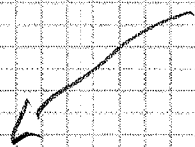
$$\ln y = \ln \left(\frac{x^3 (3-x^2)^3}{(2x+3)^2 (4-2x)^5} \right)$$

Properties of Logs

$$\log_b (X \cdot Y) = \log_b X + \log_b Y$$

$$\log_b \left(\frac{X}{Y} \right) = \log_b X - \log_b Y$$

$$\log_b x^n = n \log_b x$$



$$\ln y = \ln (x^3 \cdot (3-x^2)^3) - \ln [(2x+3)^2 (4-2x)^5]$$

$$\ln y = \ln x^3 + \ln (3-x^2)^3 - [\ln (2x+3)^2 + \ln (4-2x)^5]$$

$$\ln y = 3 \ln x + 3 \ln (3-x^2) - 2 \ln (2x+3) - 5 \ln (4-2x)$$

Do implicit diff

$$\frac{1}{y} \cdot \frac{dy}{dx} = 3 \cdot \frac{1}{x} + 3 \frac{1}{3-x^2} (-2x) - 2 \frac{1}{2x+3} \cdot 2 - 5 \cdot \frac{1}{4-2x} (-2)$$

$$\frac{dy}{dx} = \left(\frac{3}{x} - \frac{6}{3-x^2} - \frac{4}{2x+3} + \frac{10}{4-2x} \right) \cdot \frac{x^3 (3-x^2)^3}{(2x+3)^2 (4-2x)^5}$$