

LOGARITHMIC EQUATIONS

Solve for x.

$$1) \log_2 x = 3 \Rightarrow x = 2^3 = 8$$

$$2) \log_x 25 = 2 \Rightarrow x^2 = 25 \Rightarrow x = 5$$

$$3) 3 \log_2 x = \log_2 8 \Rightarrow \log_2 x^3 = \log_2 8 \Rightarrow \\ \Rightarrow x^3 = 8 \Rightarrow x = 2$$

$$4) \log_4 (x+2) + \log_4 (x-1) = 1$$

$$\log_4 [(x+2)(x-1)] = \log_4 4$$

$$(x+2)(x-1) = 4$$

$$x^2 + x - 2 = 4$$

$$x^2 + x - 6 = 0$$

$$\begin{array}{r|l} \bullet = -6 & 3, -2 \\ + = 1 & 1 \end{array}$$

$$(x+3)(x-2) = 0 \Rightarrow x+3=0 \text{ or } x-2=0$$

$$\boxed{\cancel{x = -3}}$$

$$\boxed{x = 2}$$

check

$$5) (\log_4 x)^2 \cdot \log_2 x = 2$$

change of base

$$\log_2 x = \frac{\log_4 x}{\log_4 2}$$

$$(\log_4 x)^2 \cdot \frac{\log_4 x}{\log_4 2} = 2$$

$$\log_4 2 = ?$$

$$4^? = 2$$

$$4^{1/2} = \sqrt{4} = 2$$

$$\frac{(\log_4 x)^3}{1/2} = 2$$

$$(\log_4 x)^3 = 2 \cdot \frac{1}{2}$$

$$x^3 = 1 \rightarrow x = 1$$

$$(\log_4 x)^3 = 1$$

$$\log_4 x = 1 \Rightarrow \boxed{x = 4}$$

$$6) \log_4 x + \log_5 x = 3$$

$$\frac{\ln x}{\ln 4} + \frac{\ln x}{\ln 5} = 3 \Rightarrow \ln x \left(\frac{1}{\ln 4} + \frac{1}{\ln 5} \right) = 3$$

$$\Rightarrow \ln x = \frac{3}{\frac{1}{\ln 4} + \frac{1}{\ln 5}} \Rightarrow x = e^{\frac{3}{\frac{1}{\ln 4} + \frac{1}{\ln 5}}} \approx 9.34$$