MATRIX ALGEBRA

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & -1 \end{bmatrix}_{2\times 3} \qquad B = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & 2 \end{bmatrix}_{2\times 3}$$

Addition

$$A + B = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

subtraction

$$A - B = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & -1 \end{bmatrix}_{2\times 3} - \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & 2 \end{bmatrix}_{2\times 3} = \begin{bmatrix} 1 & 0 & 5 \\ 1 & -1 & -3 \end{bmatrix}_{2\times 3}$$

$$3 - (-2) = 3 + 2 = 5$$

Multiplication of a Matrix by a Number

$$2 \cdot A = 2 \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 6 \\ 2 & 0 & -2 \end{bmatrix}$$

Multiplication of two Matrices

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & -1 \end{bmatrix}_{2\times 3} \qquad B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ -1 & 2 \end{bmatrix}_{3\times 2}$$

$$A \times B = \begin{bmatrix} 1 & 7 \\ 2 & -2 \end{bmatrix}_{2\times 2}$$

$$2\times 1 + 1\times 2 + 3(-1) = 2 + 2 - 3 = 4 - 1 = 1$$
$$2\times 0 + 1\times 1 + 3\times 2 = 0 + 1 + 6 = 7$$
$$1\times 1 + 0\times 2 + (-1)(-1) = 1 + 0 + 1 = 2$$
$$1\times 0 + 0\times 1 + (-1)2 = 0 + 0 - 2 = -2$$

$$A_{2\times 3} \times B_{3\times 1} = C_{2\times 1}$$

$A_{2\times 3} \times B_{2\times 4}$ cannot be done

$$A + B = B + A \qquad A \times B \neq B \times A$$

$$A_{2\times 3} \qquad O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2\times 3} \qquad A + O = A$$
$$O + A = A$$

Identity Matrix for Multiplication

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & -1 \end{bmatrix}_{2\times 3} \qquad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$$

$$A \times I = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & -1 \end{bmatrix}_{2\times 3}$$

$A_{m\times n} \times I_n = A_{m\times n}$ $\qquad I_m \times A_{m\times n} = A_{m\times n}$

THE INVERSE OF SQUARE MATRIX

$A_{n\times n}$ if there exists a matrix $A^{-1}_{n\times n}$

such that $A \times A^{-1} = I_n$

and $A^{-1} \times A = I_n$

then A^{-1} is called the inverse of A