SYSTEMS OF LINEAR EQUATIONS. MATRIX METHODS

$$\begin{cases} x+y+z=3 \\ x-7z=-6 \\ z=1 \end{cases}$$

Triangular Form

"Back Substitution"

$x-7\cdot 1=-6 \Rightarrow x=-6+7=1$

$x+1+1=3 \Rightarrow x+2=3 \Rightarrow x=1$

$x=1$, $y=2$, $z=3$

$$\begin{cases} 2x+3y-z=5 \\ x-y+2z=5 \\ 3x+2y-4z=-5 \end{cases}$$

NOTE

1) $2x+3y-z=5$

$x(2)$ $\quad 4x+6y-2z=10$

$4\cdot 1+6\cdot 2-2\cdot 3=10$

Multiplying an equation by a number doesn't change the solution

2) $2x+3y-z=5$

$x-y+2z=5$

$3x+2y+z=10$

$3\cdot 1+2\cdot 2+3=10$

3) $\begin{cases} 2x+3y-z=5 \\ x-y+2z=5 \\ 3x+2y-4z=-5 \end{cases}$ $\quad$ $\begin{cases} x-y+2z=5 \\ 2x+3y-z=5 \\ 3x+2y-4z=-5 \end{cases}$

Solve

$$\begin{cases} 2x + 3y - 5z = 23 \\ x + y + z = 0 \\ -3x + 4y + 2z = -1 \end{cases}$$

Three row operations

1) Interchange any two rows
2) Multiply a row by a non-zero number
3) Replace any row by the sum of itself and a multiple of another row

Augmented Matrix

$$\begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}\begin{bmatrix} 2 & 3 & -5 & 23 \\ 1 & 1 & 1 & 0 \\ -3 & 4 & 2 & -1 \end{bmatrix}$$

Goal

$$\begin{bmatrix} 1 & \square & \square & \square \\ 0 & 1 & \square & \square \\ 0 & 0 & 1 & \square \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}\begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 3 & -5 & 23 \\ -3 & 4 & 2 & -1 \end{bmatrix}$$

$R_1 \times (-2) + R_2 \to R_2$

$1 \times (-2) + 2 = 0$
$1 \times (-2) + 3 = 1$
$1 \times (-2) + -5 = -7$
$0 \times (-2) + 23 = 23$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -7 & 23 \\ -3 & 4 & 2 & -1 \end{bmatrix}$$

$R_1 \times (3) + R_3 \to R_3$

$1 \times (3) + (-3) = 0$
$1 \times (3) + 4 = 7$
$1 \times (3) + 2 = 5$
$0 \times (3) + (-1) = -1$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -7 & 23 \\ 0 & 7 & 5 & -1 \end{bmatrix}$$

$R_2 \times (-7) + R_3 \to R_3$

$0 \times (-7) + 0 = 0$

$1 \times (-7) + 7 = 0$

$-7 \times (-7) + 5 = 54$

$23 \times (-7) + (-1) = -162$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -7 & 23 \\ 0 & 0 & 54 & -162 \end{array}\right]$$

$R_3 \times \left(\frac{1}{54}\right) \to R_3$

$0 \times \left(\frac{1}{54}\right) = 0$

$0 \times \left(\frac{1}{54}\right) = 0$

$54 \times \left(\frac{1}{54}\right) = 1$

$-162 \times \left(\frac{1}{54}\right) = -3$

$$\begin{array}{c} x \quad y \quad z \\ \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -7 & 23 \\ 0 & 0 & 1 & -3 \end{array}\right] \end{array}$$

$\boxed{z = -3}$

$y - 7z = 23 \Rightarrow y - 7(-3) = 23$

$\Rightarrow y + 21 = 23 \Rightarrow \boxed{y = 2}$

$x + y + z = 0 \Rightarrow x + 2 + (-3) = 0$

$\Rightarrow x - 1 = 0 \Rightarrow \boxed{x = 1}$

Solution $(1, 2, -3)$