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SYSTEMS OF LINEAR EQUATIONS MATRIX METHODS. PART II

$$\begin{cases} 2x+3y-5z=23 \\ x+y+z=0 \\ -3x+4y+2z=-1 \end{cases} \longrightarrow \begin{cases} x+y+z=0 \\ y-7z=23 \\ z=-3 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & -5 & 23 \\ 1 & 1 & 1 & 0 \\ -3 & 4 & 2 & -1 \end{array}\right] \longrightarrow \begin{array}{c} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -7 & 23 \\ 0 & 0 & 1 & -3 \end{array}\right] \end{array} \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}$$

Plan:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \square \\ 0 & 1 & 0 & \square \\ 0 & 0 & 1 & -3 \end{array}\right]$$

$R_3 \times (7) + R_2 \to R_2$

$0 \times 7 + 0 = 0$

$0 \times 7 + 1 = 1$

$1 \times 7 + (-7) = 0$

$-3 \times 7 + 23 = 2$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{array}\right]$$

$R_3 \times (-1) + R_1 \to R_1$

$0 \times (-1) + 1 = 1$

$0 \times (-1) + 1 = 1$

$1 \times (-1) + 1 = 0$

$-3 \times (-1) + 0 = 3$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{array}\right]$$

$R_2 \times (-1) + R_1 \rightarrow R_1$

$0 \times (-1) + 1 = 1$
$1 \times (-1) + 1 = 0$
$0 \times (-1) + 0 = 0$
$2 \times (-1) + 3 = 1$

$$\begin{array}{ccc}
x & y & z \\
\end{array}
\left[\begin{array}{ccc|c}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 2 \\
0 & 0 & 1 & -3
\end{array}\right]$$

$x = 1 \quad y = 2 \quad z = -3$

Gauss-Jordan Elimination Method

Note:

$$\left[\begin{array}{cccc|c}
1 & \square & \square & \square & \square \\
0 & 0 & 1 & \square & \square \\
0 & 0 & 0 & 1 & \square
\end{array}\right]$$

Row-Echelon Form

1) the first non-zero entry in each row is 1
2) the number of leading zeros in any row is greater than the number of leading zeros in previous rows
3) If there are rows with all zero entries, they are at the bottom of the matrix

$$\left[\begin{array}{cccc|c}
1 & \square & \square & \square & \square \\
0 & 0 & \square & \square & \square \\
0 & \square & \square & \square & \square
\end{array}\right]$$

If the row-echelon form of the augmented matrix has a row of the form

$$\left[\begin{array}{cccc|c} 0 & 0 & \cdots & 0 & \square \end{array}\right]$$

the system is inconsistent

If the row-echelon form of the augmented matrix has a row of the form

$$\left[\begin{array}{cccc|c} 0 & 0 & \cdots & 0 & 0 \end{array}\right]$$

that row can be eliminated

If the row-echelon form of the augmented matrix is triangular

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \square \\ 0 & 1 & 0 & \square \\ 0 & 0 & 1 & \square \end{array}\right]$$

the system has a unique solution

If the row-echelon form of the augmented matrix is something like this:

$$\begin{array}{ccccc} x & y & z & w & \\ \end{array}$$
$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & 3 & 6 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right]$$

Reduced row-echelon Form

$z + w = 2 \Rightarrow \boxed{z = 2 - w}$

$y + 3w = 6 \Rightarrow \boxed{y = 6 - 3w}$

$x - w = 3 \Rightarrow \boxed{x = 3 + w}$

Free variable

$(3 + w, 6 - 3w, 2 - w, w)$