SYSTEMS OF LINEAR EQUATIONS (PART IV)

Method of Elimination

Example:

$$\begin{cases} 2x+3y=5 & (\times 2) \\ 4x-2y=2 & (\times 3) \end{cases} \quad + \quad \begin{array}{r} 4x+6y=10 \\ 12x-6y=6 \\ \hline 16x=16 \Rightarrow \boxed{x=1} \end{array}$$

$$2x+3y=5 \Rightarrow 2\times 1+3y=5 \Rightarrow 3y=3 \Rightarrow \boxed{y=1}$$

Example

$$\begin{cases} 2x+3y=5 & (\times 2) \\ 4x+6y=7 & \end{cases} \quad - \quad \begin{array}{r} 4x+6y=10 \\ 4x+6y=7 \\ \hline 0=3 \end{array} \quad \text{ABSURD}$$

Inconsistent System
No Solution

Example

$$\begin{cases} 2x+3y=5 & (\times 2) \\ 4x+6y=10 & \end{cases} \quad - \quad \begin{array}{r} 4x+6y=10 \\ 4x+6y=10 \\ \hline 0=0 \end{array} \quad \text{TRUE}$$

Dependent
Infinite Solutions

$$2x=5-3y \Rightarrow x=\frac{5-3y}{2}$$

solution: $\left(\frac{5-3y}{2}, y\right)$

or $3y=5-2x \Rightarrow y=\frac{5-2x}{3}$

solution $\left(x, \frac{5-2x}{3}\right)$

Larger Systems

$$\begin{cases} 2x+3y+4z=20 \\ x-y+z=2 \\ 3x+2y-5z=-8 \end{cases}$$

$$\begin{array}{r} 2x+3y+4z=20 \\ -\ x-y+z=2 \end{array} \quad (\times 4) \quad \begin{array}{r} 2x+3y+4z=20 \\ -\ 4x-4y+4z=8 \end{array}$$

$$-2x+7y=12$$

$$\begin{array}{r} x-y+z=2 \\ 3x+2y-5z=-8 \end{array} \quad (\times 5) \quad \begin{array}{r} 5x-5y+5z=10 \\ +\ 3x+2y-5z=-8 \end{array}$$

$$8x-3y=2$$

$$\begin{cases} -2x+7y=12 \\ 8x-3y=2 \end{cases} \quad (\times 4) \quad \begin{array}{r} -8x+28y=48 \\ +\ 8x-3y=2 \end{array}$$

$$25y=50 \Rightarrow \boxed{y=2}$$

$$8x-3*2=2 \Rightarrow 8x=8 \Rightarrow \boxed{x=1}$$

$$x-y+z=2 \Rightarrow 1-2+z=2 \Rightarrow z=2-1+2 \Rightarrow \boxed{z=3}$$

Example

$$\begin{cases} 2x+3y+4z=20 \\ 4x+6y+8z=15 \\ 3x+2y-5z=-8 \end{cases}$$

$$\begin{array}{r} 2x+3y+4z=20 \\ 4x+6y+8z=15 \end{array} \quad (\times 2) \quad \begin{array}{r} 4x+6y+8z=40 \\ -\ 4x+6y+8z=15 \end{array}$$

$0=25$ ABSURD

System is inconsistent
No solution

example

$$\begin{cases} 2x+3y+4z=20 \\ 4x+6y+8z=40 \\ 3x+2y-5z=-8 \end{cases}$$

$2x+3y+4z=20$ $(\times 2)$ $\quad 4x+6y+8z=40$

$-\ 4x+6y+8z=40$ $\quad -\ 4x+6y+8z=40$

$0=0$ TRUE

$2x+3y+4z=20$ $(\times 5)$ $\quad 10x+15y+20z=100$

$3x+2y-5z=-8$ $(\times 4)$ $+\ 12x+8y-20z=-32$

$22x+23y=68$

$$\begin{cases} 2x+3y+4z=20 \\ 22x+23y=68 \end{cases} \quad \begin{cases} 2x+3y=20-4z \\ 22x+23y=68 \end{cases} \quad (\times 11)$$

$22x+33y=220-44z$

$-\ 22x+23y=68$

$10y=152-44z \Rightarrow y=\dfrac{152-44z}{10}=\dfrac{76-22z}{5}$

$22x+23\left(\dfrac{76-22z}{5}\right)=68 \Rightarrow 22x+\dfrac{1748-506z}{5}=68$

$\Rightarrow 22x=68-\dfrac{1748-506z}{5}=\dfrac{340-1748+506z}{5}$

$22x=\dfrac{-1408+506z}{5} \Rightarrow x=\dfrac{-1408+506z}{5\times 22}$

$x=\dfrac{-64+23z}{5}$

Solution: $\left(\dfrac{-64+23z}{5},\ \dfrac{76-22z}{5},\ z\right)$