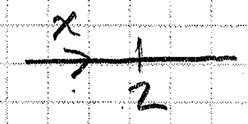
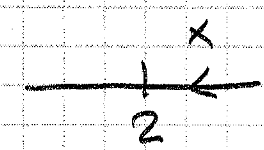
MORE LIMITS

Ex: $\lim\limits_{x\to 2^-} \dfrac{\overbrace{2x+3}^{+} \to 7}{\underbrace{x-2}_{-} \to 0} = -\infty$



Ex: $\lim\limits_{x\to 2^+} \dfrac{2x+3 \to 7}{\underbrace{x-2}_{+} \to 0} = +\infty$



Ex: $\lim\limits_{x\to 2} \dfrac{2x+3}{x-2}$ does not exist (d.n.e.)

NOTE:

$\dfrac{12}{4} = 3$ because $3 \cdot 4 = 12$

$\dfrac{0}{4} = 0$ " $0 \cdot 4 = 0$

$\dfrac{12}{0} = ?$ undefined

$\dfrac{0}{0} = 0$ $\quad \dfrac{0}{0} = 6.7$ $\quad \dfrac{0}{0} = 3.4$ $\quad \dfrac{0}{0}$ indetermination

We must try to "eliminate" the indetermination

Ex: $\lim\limits_{x\to 2} \dfrac{2-x \to 0}{\underset{4 \;\; -10 \; +6}{x^2-5x+6} \to 0} = \lim\limits_{x\to 2} \dfrac{(2-x)}{(x-2)(x-3)} =$

$\cdot = 6$ | $-2, -3$ |
$+ = -5$ | ✓ |

TRICK: $2-x = (-1)(x-2)$

$= \lim\limits_{x\to 2} \dfrac{(-1)\cancel{(x-2)}}{\cancel{(x-2)}(x-3)} = \lim\limits_{x\to 2} \dfrac{-1}{x-3} = \dfrac{-1}{2-3} = 1$

FACTORING: TRICK #1

EX: $\lim\limits_{x\to 9} \dfrac{\overset{\to 0}{x-9}}{\underset{\to 0}{\sqrt{x}-3}}$

TRICK #2: Multiply by the "conjugate" of the expression that contains the square root

the conjugate of $a+b$ is $a-b$

$(a+b)(a-b) = a^2 - \cancel{ab} + \cancel{ab} - b^2 = a^2 - b^2$

$(\sqrt{x}-3)(\sqrt{x}+3) = x-9$

warning $(\sqrt{x})^2 = \sqrt{x^2} = |x|$

$$\lim_{x\to 9}\frac{(x-9)\cdot(\sqrt{x}+3)}{(\sqrt{x}-3)\cdot(\sqrt{x}+3)} = \lim_{x\to 9}\frac{\cancel{(x-9)}(\sqrt{x}+3)}{\cancel{(x-9)}}$$

$$\lim_{x\to 9}\frac{\sqrt{x}+3}{1} = \sqrt{9}+3 = 3+3 = 6$$

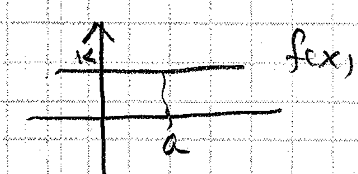
EX: $\lim\limits_{x\to+\infty}(\underbrace{3x^2}_{\to+\infty}\underbrace{-5x}_{\to-\infty}+9) = \lim\limits_{x\to+\infty}(3x^2) = +\infty$

who "dominates"? $3x^2$ dominates

EX: $\lim\limits_{x\to-\infty}\dfrac{3x^2-5x+9}{2x^2-4x+100} = \lim\limits_{x\to-\infty}\dfrac{3\cancel{x^2}}{2\cancel{x^2}} =$

$= \lim\limits_{x\to-\infty}\left(\frac{3}{2}\right) = 3/2$

PROPERTY $\lim\limits_{x\to a}(K) = K$



Ex: $\lim_{x \to -\infty} \frac{3x^2 - 5x + 9}{2x^3 - 4x + 100} = \lim_{x \to -\infty} \frac{3x^2}{2x^3} =$

$= \lim_{x \to -\infty} \frac{3}{2x} = \frac{3}{2} \lim_{x \to -\infty} \frac{1}{x} = \frac{3}{2} \cdot 0 = 0$

PROPERTY $\lim_{x \to a} (k \cdot f(x)) = k \lim_{x \to a} f(x)$