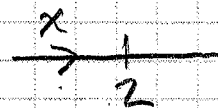


MORE LIMITS

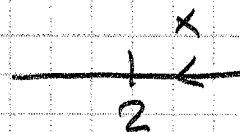
EX: $\lim_{x \rightarrow 2^-} \frac{2x+3}{x-2} = -\infty$

(Note: In the original image, the numerator $2x+3$ is marked with a '+' and the denominator $x-2$ is marked with a '-' as x approaches 2 from the left.)



EX: $\lim_{x \rightarrow 2^+} \frac{2x+3}{x-2} = +\infty$

(Note: In the original image, the numerator $2x+3$ is marked with a '+' and the denominator $x-2$ is marked with a '+' as x approaches 2 from the right.)



EX: $\lim_{x \rightarrow 2} \frac{2x+3}{x-2}$ does not exist (d.n.e.)

NOTE:

$\frac{12}{4} = 3$ because $3 \cdot 4 = 12$

$\frac{0}{4} = 0$ " $0 \cdot 4 = 0$

$\frac{12}{0} = ?$ undefined

$\frac{0}{0} = 0$ $\frac{0}{0} = 67$ $\frac{0}{0} = 3.4$ $\frac{0}{0}$ indetermination

we must try to "eliminate" the indetermination

EX: $\lim_{x \rightarrow 2} \frac{2-x}{x^2-5x+6} = \lim_{x \rightarrow 2} \frac{(2-x)}{(x-2)(x-3)} =$

(Note: In the original image, the denominator x^2-5x+6 is factored into $(x-2)(x-3)$ with the numbers 4, -10, and 6 written below it.)

$\frac{0}{0} = 6$ | $-2-3$ | TRICK: $2-x = (-1)(x-2)$
 $\frac{+}{+} = -15$ | \checkmark |

$= \lim_{x \rightarrow 2} \frac{(-1)(\cancel{x-2})}{(\cancel{x-2})(x-3)} = \lim_{x \rightarrow 2} \frac{-1}{x-3} = \frac{-1}{2-3} = 1$

FACTORIZING: TRICK #1

$$\text{EX: } \lim_{x \rightarrow 9} \frac{\cancel{x-9} \rightarrow 0}{\cancel{\sqrt{x}-3} \rightarrow 0}$$

TRICK #2: Multiply by the "conjugate" of the expression that contains the square root
The conjugate of $a+b$ is $a-b$

$$(a+b)(a-b) = a^2 - \cancel{ab} + \cancel{ab} - b^2 = a^2 - b^2$$

$$(\sqrt{x}-3)(\sqrt{x}+3) = x-9$$

$$\text{warning } (\sqrt{x})^2 = \sqrt{x^2} = |x|$$

$$\lim_{x \rightarrow 9} \frac{(x-9) \cdot (\sqrt{x}+3)}{(\sqrt{x}-3) \cdot (\sqrt{x}+3)} = \lim_{x \rightarrow 9} \frac{\cancel{(x-9)} (\sqrt{x}+3)}{\cancel{(x-9)}}$$

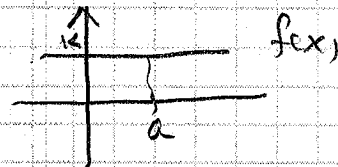
$$\lim_{x \rightarrow 9} \frac{\sqrt{x}+3}{1} = \sqrt{9}+3 = 3+3 = 6$$

$$\text{EX: } \lim_{x \rightarrow +\infty} (3x^2 - 5x + 9) = \lim_{x \rightarrow +\infty} (3x^2) = +\infty$$

who "dominates"? $3x^2$ dominates

$$\begin{aligned} \text{EX: } \lim_{x \rightarrow -\infty} \frac{3x^2 - 5x + 9}{2x^2 - 4x + 100} &= \lim_{x \rightarrow -\infty} \frac{3x^2}{2x^2} = \\ &= \lim_{x \rightarrow -\infty} \left(\frac{3}{2}\right) = \frac{3}{2} \end{aligned}$$

$$\text{PROPERTY } \lim_{x \rightarrow a} (k) = k$$



$$\text{ex: } \lim_{x \rightarrow -\infty} \frac{3x^2 - 5x + 9}{2x^3 - 4x + 100} = \lim_{x \rightarrow -\infty} \frac{3x^2}{2x^3} =$$

$$= \lim_{x \rightarrow -\infty} \frac{3}{2x} = \frac{3}{2} \lim_{x \rightarrow -\infty} \frac{1}{x} = \frac{3}{2} \cdot 0 = 0$$

PROPERTY $\lim_{x \rightarrow a} (k \cdot f(x)) = k \lim_{x \rightarrow a} f(x)$