

## Multiple Regression Analysis

Simple Linear Regression:  $y = \beta_0 + \beta_1 x + \epsilon$

Multiple Regression:  $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$

$y$  is the dependent variable

$x_1, x_2$  are the independent variables

$\beta_0, \beta_1$  or  $\beta_2$  are the regression coefficients

the least squares prediction equation is:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 \quad \text{where}$$

$\hat{y}$  is the predicted value of  $y$ , and

$\hat{\beta}_0, \hat{\beta}_1$  and  $\hat{\beta}_2$  are the estimated coefficients

the independent variables can be functions of other independent variables:

EX:  $x_2 = x_1^3$

$x_3 = x_1 * x_2$

higher-order terms

Later Videos

A model without higher-order terms is called a first-order model

the independent variables can be quantitative or qualitative

IN THIS VIDEO:

## FIRST-ORDER MODEL WITH QUANTITATIVE VARIABLES

Least-squares method

which minimizes  $SSE = \sum (y - \hat{y})^2$

SPSS, stata, R

SAS, Minitab

Example:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$$

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2$$

y	x <sub>1</sub>	x <sub>2</sub>
5	1	50
8	2	51
14	3	34
23	4	22
45	5	15
48	6	10

REGRESSION  
 /MISSING LISTWISE  
 /STATISTICS COEFF OUTS R ANOVA  
 /CRITERIA=PIN(.05) POUT(.10)  
 /NOORIGIN  
 /DEPENDENT y  
 /METHOD=ENTER x1 x2.

**Variables Entered/Removed<sup>a</sup>**

Model	Variables Entered	Variables Removed	Method
1	x2, x1 <sup>b</sup>		Enter

a. Dependent Variable: y

b. All requested variables entered.

**Model Summary**

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.962 <sup>a</sup>	.925	.875	6.585

a. Predictors: (Constant), x2, x1

**ANOVA<sup>a</sup>**

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	1604.746	2	802.373	18.504	.021 <sup>b</sup>
	Residual	130.088	3	43.363		
	Total	1734.833	5			

a. Dependent Variable: y

b. Predictors: (Constant), x2, x1

**Coefficients<sup>a</sup>**

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	-1.173	45.611		-.026	.981
	x1	8.326	6.814	.836	1.222	.309
	x2	-.136	.725	-.129	-.188	.863

a. Dependent Variable: y