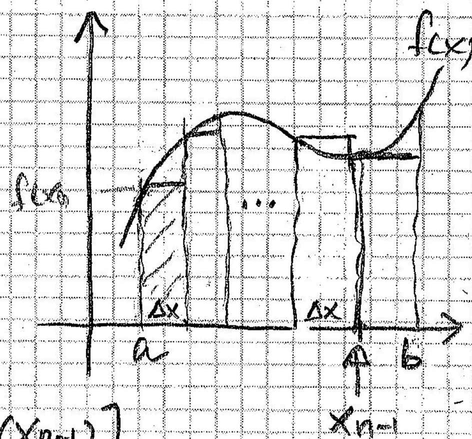
NUMERICAL INTEGRATION

$$\int_a^b f(x)\,dx$$

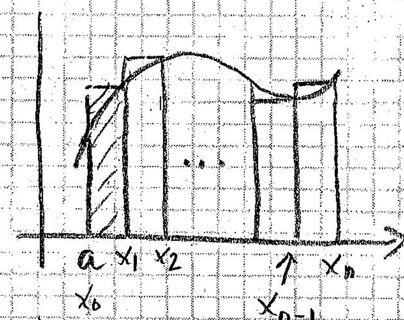


$$\Delta x = \frac{b-a}{n}$$

Left-End $\quad L_n = \Delta x\, [f(x_0) + f(x_1) + \cdots f(x_{n-1})]$

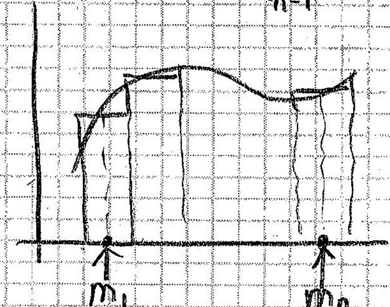
Right-End

$$R_n = \Delta x\, (f(x_1) + f(x_2) + \cdots + f(x_n))$$



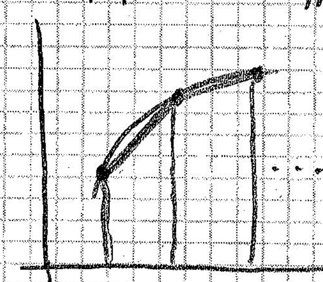
Mid-Point

$$M_n = \Delta x\, (f(m_1) + f(m_2) + \cdots + f(m_n))$$



the Trapezoidal Approximation

$$T_n = \frac{1}{2}\Delta x\, [f(x_0) + 2f(x_1) + 2f(x_2) + \cdots$$
$$\cdots + 2f(x_{n-1}) + f(x_n)]$$



Simpson's Rule $\quad$ (Parabolas)

$$S_n = \frac{\Delta x}{3}\, [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots$$
$$\cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

1, 4, 2, 4, 2, . . . , 4, 2, 4, 1

n is <u>even</u>

Example

$$\int_1^2 \frac{1}{x}\,dx = \int_1^2 x^{-1}\,dx = \ln|x|\Big|_1^2 =$$

$$\ln 2 - \cancelto{0}{\ln 1} = 0.69314718\,1$$

$n = 10 \qquad \Delta x = \frac{2-1}{10} = 0.1$

1.0 1.1 1.2 1.3 ... 1.9 2.0

$$L_n = \Delta x \left(f(x_0) + f(x_1) + \cdots + f(x_9) \right)$$

$$= 0.1 \left(\frac{1}{1} + \frac{1}{1.1} + \cdots + \frac{1}{1.9} \right) \approx .718771$$

$$R_n = \Delta x \left(f(x_1) + f(x_2) + \cdots + f(x_n) \right) =$$

$$= 0.1 \left(\frac{1}{1.1} + \frac{1}{1.2} + \cdots + \frac{1}{2} \right) \approx .668771$$

$$M_n = \Delta x \left(f(m_1) + f(m_2) + \cdots + f(m_{10}) \right)$$

$$= 0.1 \left(f(1.05) + f(1.15) + \cdots + f(1.95) \right)$$

$$= 0.1 \left(\frac{1}{1.05} + \frac{1}{1.15} + \cdots + \frac{1}{1.95} \right) \approx .692835$$

$$T_n = \frac{1}{2} \Delta x \left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n) \right]$$

$$= \frac{1}{2} * 0.1 \left[\frac{1}{1} + 2 * \frac{1}{1.1} + 2 * \frac{1}{1.2} + \cdots + 2 * \frac{1}{1.9} + \frac{1}{2} \right] \approx .693771$$

$$S_n = \frac{\Delta x}{3} \left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + 4f(x_5) \right.$$

$$\left. + 2f(x_6) + 4f(x_7) + 2f(x_8) + 4f(x_9) + f(x_{10}) \right]$$

$$= \frac{0.1}{3} \left[\frac{1}{1} + 4 * \frac{1}{1.1} + 2 * \frac{1}{1.2} + \cdots + 4 * \frac{1}{1.9} + \frac{1}{2} \right] \approx .693150$$