INTEGRATING RATIONAL FUNCTIONS BY PARTIAL FRACTIONS

$$R(x) = \frac{P(x)}{Q(x)}$$

Ex: $R(x) = \dfrac{2x+1}{x^2 - 3x}$ proper

Ex: $R(x) = \dfrac{2x^3+5x^2+x+4}{x+1}$ improper

$$\frac{3}{x+1} - \frac{2}{x-1} = \frac{3(x-1) - 2(x+1)}{(x+1)(x-1)} = \frac{x-5}{(x+1)(x-1)} \quad \text{proper}$$

Ex: $I = \displaystyle\int \frac{2x^3+5x^2+x+4}{x+1}\,dx$

$$\begin{array}{r|l}
 & 2x^2 + 3x - 2 \\ \hline
x+1 & 2x^3 + 5x^2 + x + 4 \\
 & -(2x^3 + 2x^2) \\ \hline
 & 3x^2 + x \\
 & -(3x^2 + 3x) \\ \hline
 & -2x + 4 \\
 & -(-2x - 2) \\ \hline
 & 2
\end{array}$$

$\dfrac{2x^3}{x} = 2x^2$

$\dfrac{3x^2}{x} = 3x$

$-\dfrac{2x}{x} = -2$

$$\frac{2x^3+5x^2+x+4}{x+1} = 2x^2 + 3x - 2 + \frac{2}{x+1}$$

$$I = 2\int x^2\,dx + 3\int x\,dx - 2\int dx + 2\int \frac{dx}{x+1}$$

$$= 2\frac{x^3}{3} + 3\frac{x^2}{2} - 2x + 2\ln|x+1| + C$$

Case 1 $R(x) = \dfrac{P(x)}{Q(x)}$

$Q(x)$ is a product of distinct linear factors

Ex: $I = \displaystyle\int \frac{x^2+3x-2}{x^3-2x^2-3x}\,dx$

$x^3-2x^2-3x = x(x^2-2x-3) = x(x+1)(x-3)$

$$\frac{x^2+3x-2}{x(x+1)(x-3)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-3}$$

$$= \frac{A(x+1)(x-3) + B\,x(x-3) + C\,x(x+1)}{x(x+1)(x-3)}$$

$x=-1 \quad -4 = 4B \Rightarrow B = -1$

$x=3 \quad 16 = 12C \Rightarrow C = \frac{16}{12} = 4/3$

$x=0 \quad -2 = -3A \Rightarrow A = 2/3$

$$I = \frac{2}{3}\int \frac{dx}{x} - \int \frac{dx}{x+1} + \frac{4}{3}\int \frac{dx}{x-3}$$

$$= \frac{2}{3}\ln|x| - \ln|x+1| + \frac{4}{3}\ln|x-3| + C \quad \text{Done!}$$

Next videos Case 2, Case 3, Case 4