INTEGRATING RATIONAL FUNCTIONS BY PARTIAL FRACTIONS II

Case 2 $R(x) = \dfrac{P(x)}{Q(x)}$

$Q(x)$ is a product of linear factors some of which are repeated

Ex: $\dfrac{x^2+3x+2}{x^2(x+1)^3} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+1} + \dfrac{D}{(x+1)^2} + \dfrac{E}{(x+1)^3}$

Ex: $I = \displaystyle\int \frac{x^4-1}{x^3-3x+2}\,dx$

$x^3-3x+2 \;\big|\; x^4+0x^3+0x^2+0x-1$

$x^4 \qquad -3x^2+2x$

$3x^2-2x$

$= \displaystyle\int \left(x + \frac{3x^2-2x}{x^3-3x+2}\right) dx$

$= \displaystyle\int x\,dx + \int \frac{3x^2-2x}{(x-1)^2(x+2)}\,dx$

	1	0	−3	2
1		1	1	−2
	1	1	−2	0
1		1	2	
	1	2	0	

$x+2=0$

$\dfrac{3x^2-2x}{(x+2)(x-1)^2} = \dfrac{A}{x+2} + \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2}$

$= \dfrac{A(x-1)^2 + B(x+2)(x-1) + C(x+2)}{(x+2)(x-1)^2}$

$= \dfrac{A(x^2-2x+1) + B(x^2+x-1) + C(x+2)}{(x+2)(x-1)^2}$

$= \dfrac{x^2(A+B) + x(-2A+B+C) + A-B+2C}{(x+2)(x-1)^2}$

$$\begin{cases} A+B=3 \\ -2A+B+C=-2 \\ A-2B+2C=0 \end{cases} \Rightarrow \quad A=\frac{16}{9}, \quad B=\frac{11}{9}, \quad C=\frac{1}{3}$$

$$I=\int x\,dx+\int \frac{3x^2-2x}{(x-1)^2(x+2)}\,dx$$

$$=\int x\,dx+\int\left(\frac{A}{x+2}+\frac{B}{x-1}+\frac{C}{(x-1)^2}\right)dx$$

$$=\int x\,dx+\frac{16}{9}\int\frac{dx}{x+2}+\frac{11}{9}\int\frac{dx}{x-1}+\frac{1}{3}\int\frac{dx}{(x-1)^2}$$

$$=\frac{x^2}{2}+\frac{16}{9}\ln|x+2|+\frac{11}{9}\ln|x-1|-\frac{1}{3}\,\frac{1}{x-1}+C$$

NEXT VIDEO: Case 3