INTEGRATING RATIONAL FUNCTIONS BY PARTIAL FRACTIONS III

Case 3 $R(X) = \frac{P(X)}{Q(X)}$

Q(x) contains <u>irreducible</u> <u>quadratic</u> factors, none of chich is repeated

Ex: $\frac{x}{(x-1)(x^2+2)(x^2+3)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+2} + \frac{Dx+E}{x^2+3}$

Ex: $I = \int \frac{x}{(x-1)(x^2+2)}\,dx = \int \left(\frac{A}{x-1} + \frac{Bx+C}{x^2+2}\right)dx$

$$\frac{x}{(x-1)(x^2+2)} = \frac{A(x^2+2)+(Bx+C)(x-1)}{(x-1)(x^2+2)}$$

$$= \frac{Ax^2+2A+Bx^2+Cx-Bx-C}{(x-1)(x^2+2)}$$

$$\frac{0x^2+x+0}{(x-1)(x^2+2)} = \frac{x^2(A+B)+x(C-B)+2A-C}{(x-1)(x^2+2)}$$

$$\begin{cases} A+B=0 \\ C-B=1 \\ 2A-C=0 \end{cases} \Longrightarrow A = 1/3 \quad B = -1/3 \quad C = 2/3$$

$$I = \int\left(\frac{A}{x-1} + \frac{Bx+C}{x^2+2}\right)dx = \frac{1}{3}\int\frac{dx}{x-1} + \int\frac{-1/3\,x+2/3}{x^2+2}\,dx$$

$$= \frac{1}{3}\int\frac{dx}{x-1} - \frac{1}{3}\int\frac{x}{x^2+2}\,dx + \frac{2}{3}\int\frac{dx}{x^2+2}$$

$$I = \frac{1}{3}\int \frac{dx}{x-1} - \frac{1}{3}\underbrace{\int \frac{x}{x^2+2}\,dx}_{I_2} + \frac{2}{3}\underbrace{\int \frac{dx}{x^2+2}}_{I_3}$$

$$I_2 = \int \frac{x}{x^2+2}\,dx = \frac{1}{2}\int \frac{2x}{x^2+2}\,dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C_1$$

$u = x^2+2$

$du = 2x\,dx$

$$= \frac{1}{2}\ln(x^2+2) + C_1$$

$$I_3 = \int \frac{dx}{x^2+2} = \int \frac{\frac{1}{2}\,dx}{\frac{x^2}{2}+\frac{2}{2}} = \frac{1}{2}\int \frac{dx}{\left(\frac{x}{\sqrt{2}}\right)^2+1}$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + C$$

$u = \frac{1}{\sqrt{2}}x$

$du = \frac{1}{\sqrt{2}}\,dx$

$$= \frac{1}{2}\cdot\sqrt{2}\int \frac{\frac{1}{\sqrt{2}}\,dx}{\left(\frac{x}{\sqrt{2}}\right)^2+1} = \frac{\sqrt{2}}{2}\int \frac{du}{u^2+1} = \frac{1}{\sqrt{2}}\tan^{-1}u + C_2$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C_2$$

$$I = \frac{1}{3}\ln|x-1| + \frac{1}{2}\ln(x^2+2) + \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$$