INTEGRATING RATIONAL FUNCTIONS BY PARTIAL FRACTIONS IV

Case 4: $R(x) = \dfrac{P(x)}{Q(x)}$

$Q(x)$ contains a repeated, irreducible, quadratic factor

Ex: $I = \displaystyle\int \frac{6x^4+8x^3+32x^2+40x+18}{(x+2)(x^2+3)^2}\,dx$

$$\frac{6x^4+8x^3+32x^2+40x+18}{(x+2)(x^2+3)^2} = \frac{A}{x+2} + \frac{Bx+C}{x^2+3} + \frac{Dx+E}{(x^2+3)^2}$$

multiplying both sides by $(x+2)(x^2+3)^2$ we get

$$6x^4+8x^3+32x^2+40x+18 =$$
$$= A(x^2+3)^2 + (Bx+C)(x+2)(x^2+3) + (Dx+E)(x+2)(x^2+3)$$
$$= (A+B)x^4 + (2B+C)x^3 + (6A+3B+2C+D)x^2 +$$
$$+ (6B+3C+2D+E)x + (9A+6C+2E)$$

$$\begin{cases} A+B=6 \\ 2B+C=8 \\ 6A+3B+2C+D=32 \\ 6B+3C+2D+E=40 \\ 9A+6C+2E=18 \end{cases}$$

For $x=-2$

$98 = 49A \rightarrow A=2$

$$\begin{cases} B=4 \\ 2B+C=8 \\ 3B+2C+D=20 \\ 6B+3C+2D+E=40 \\ 6C+2E=0 \end{cases}$$

$A=2 \quad B=4 \quad C=0 \quad D=8 \quad E=0$

$$I = \int \left[\frac{2}{x+2} + \frac{4x}{x^2+3} + \frac{8x}{(x^2+3)^2}\right]dx =$$

$$2\,Ln|x+2| + 4\,Ln(x^2+3) - \frac{8}{x^2+3} + C$$

that should be 4

that should be 2