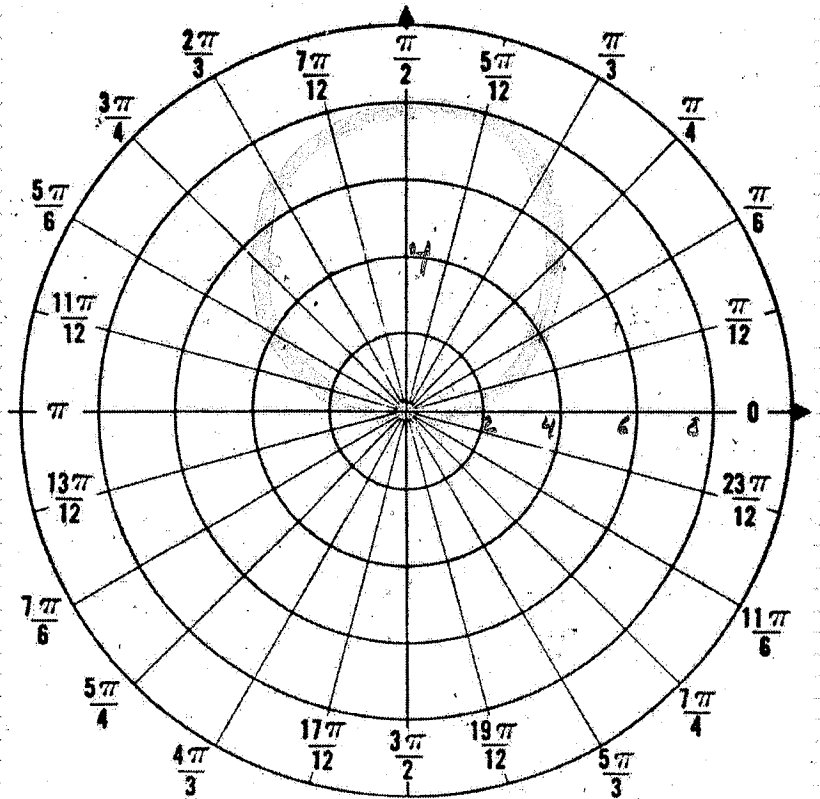
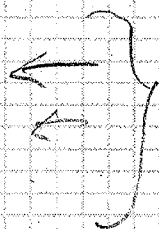


POLAR EQUATIONS AND GRAPHS

Example

$$r = 8 \sin \theta$$

θ	r
0	0
$\pi/6$	4
$\pi/4$	5.6
$\pi/3$	6.9
$\pi/2$	8
$2\pi/3$	6.9
$3\pi/4$	5.6
$5\pi/6$	4
π	0



$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$x^2 + y^2 = r^2$$

$$\theta = \tan^{-1} \left(\frac{y}{x} \right) \quad \theta = \sin^{-1} \left(\frac{y}{r} \right)$$

$$r^2 = 8 r \sin \theta \Rightarrow x^2 + y^2 = 8 y \Rightarrow x^2 + y^2 - 8 y = 0$$

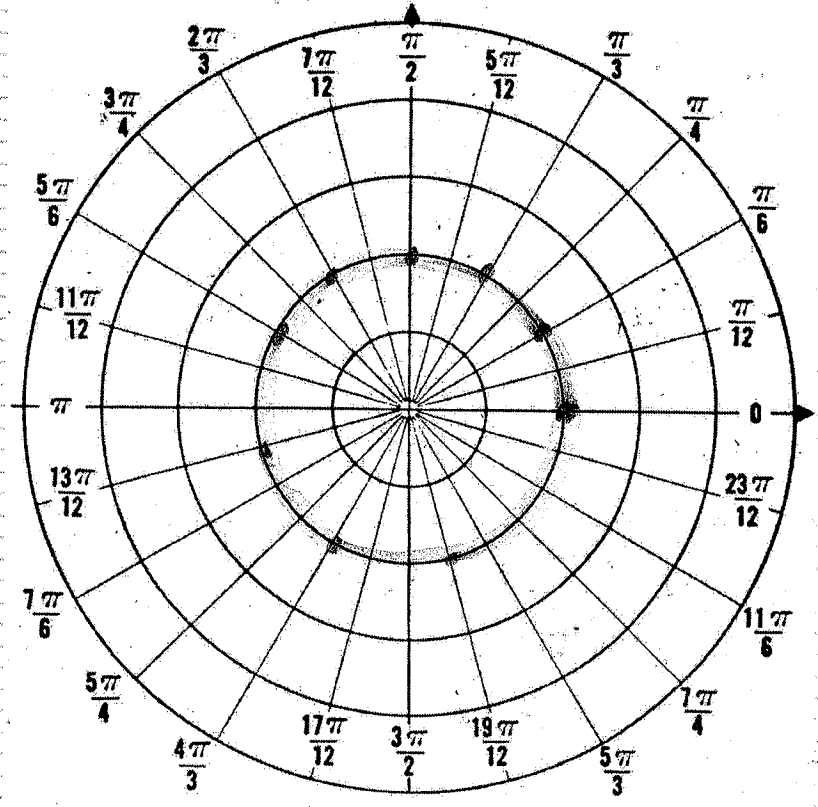
$$\Rightarrow x^2 + y^2 - 8y + 16 = 0 + 16 \Rightarrow x^2 + (y - 4)^2 = 16$$

help \rightarrow square it

$$(x - 0)^2 + (y - 4)^2 = 4^2$$

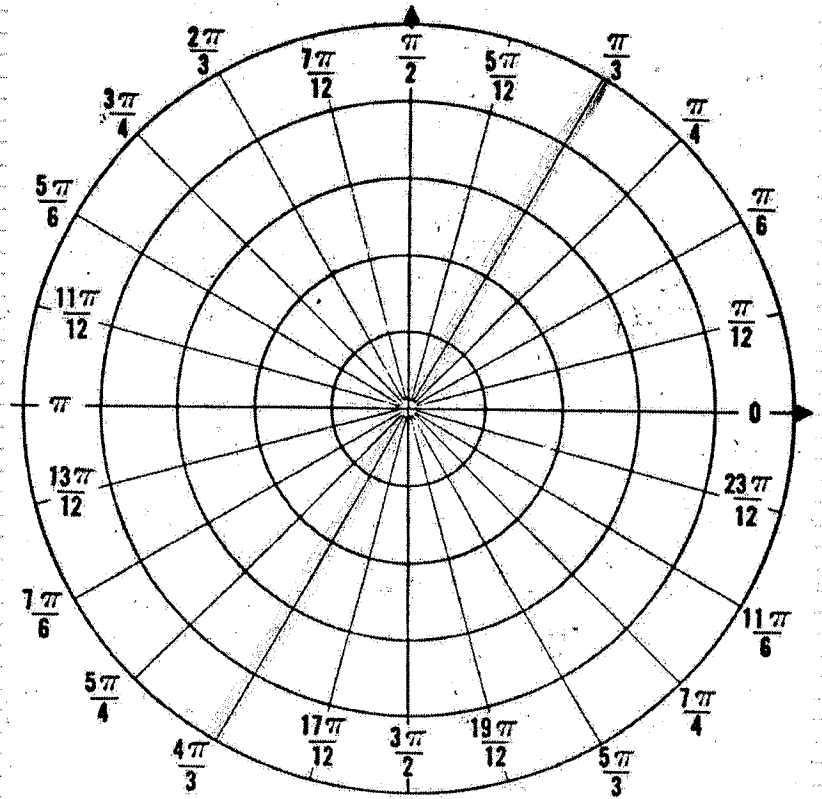
Example

$r = 2$



Ex:

$$\theta = \pi/3$$



Ex:

$$r \sin \theta = 4$$

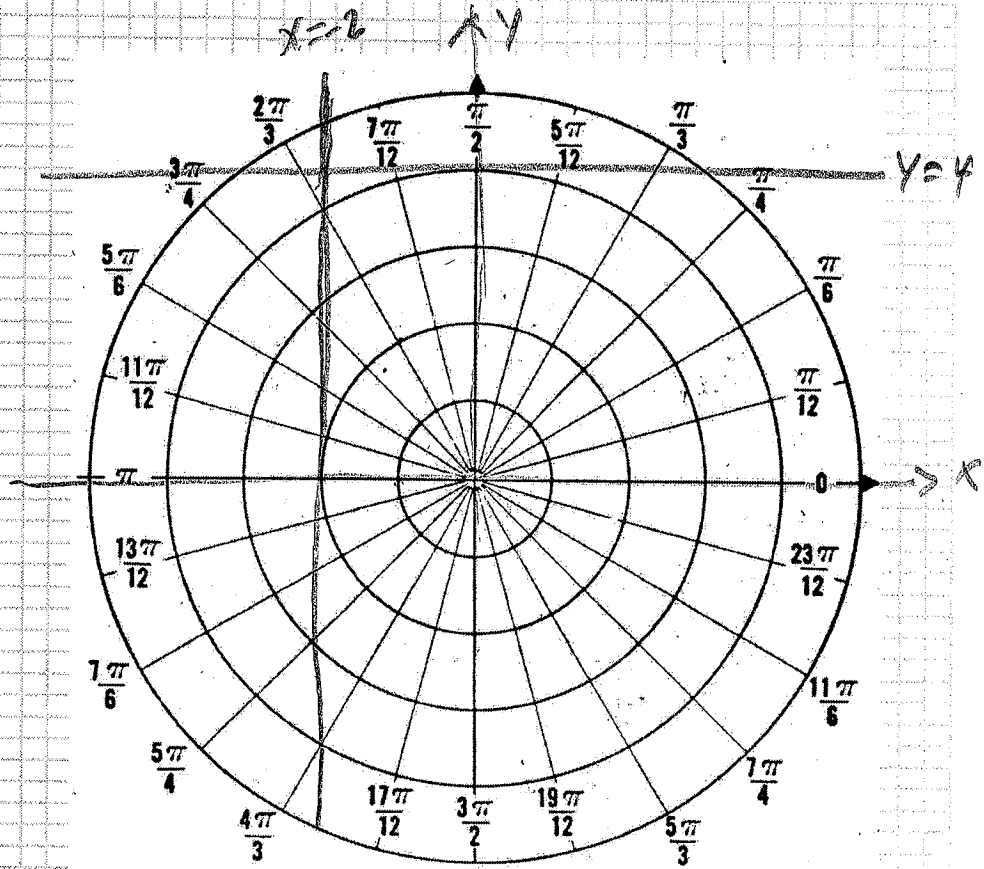
$$y = r \sin \theta$$

$$y = 4$$

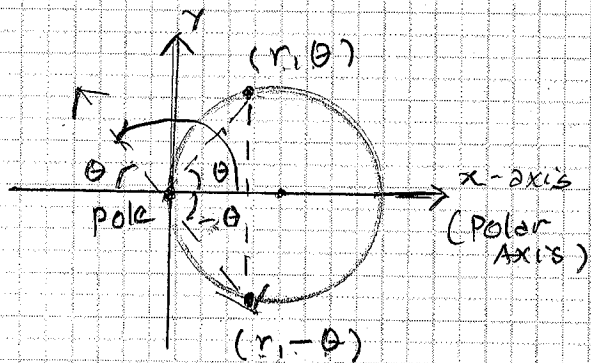
Ex:

$$r \cos \theta = -2$$

$$x = -2$$



Symmetry



the points (r, θ) and $(r, -\theta)$ are symmetric with respect to the polar axis

the point $(r, -\theta)$ can also be written as $(-r, \pi - \theta)$

Symmetry about the Polar Axis

If the point (r, θ) lies on the graph then either the point $(r, -\theta)$ or $(-r, \pi - \theta)$ lies on the graph

EX: $r = \sin(2\theta)$

$(r, -\theta)$ $r = \sin(2(-\theta)) = \sin(-2\theta) = -\sin(2\theta)$

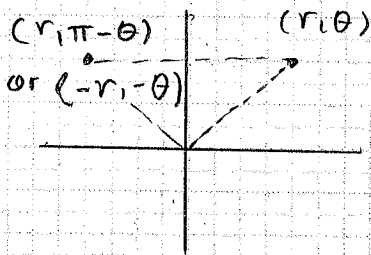
X $r = -\sin(2\theta)$

$(-r, \pi - \theta)$ $-r = \sin(2(\pi - \theta)) = \sin(2\pi - 2\theta)$

$= \sin(2\pi) \cos(2\theta) - \cos(2\pi) \sin(2\theta)$

$-r = -\sin(2\theta)$

$r = \sin(2\theta)$



About the line $\theta = \pi/2$ (y-axis)

If the point (r, θ) lies on the graph then either $(r, \pi - \theta)$ or $(-r, -\theta)$ lies on the graph

About the Pole (origin)

If the point (r, θ) lies on the graph then either $(-r, \theta)$ or $(r, \theta + \pi)$ lies on the graph

