POLYNOMIAL DIVISION AND SYNTHETIC DIVISION

$$\begin{array}{r|l} & \;\;78 \\ \hline 6 & 473 \\ & -42 \\ \hline & \;\;\;53 \\ & -48 \\ \hline & \;\;\;\;5 \end{array}$$

$$\begin{array}{r|l} & \text{quotient} \\ \hline \text{divisor} & \text{Dividend} \\ & \underbrace{}_{\text{Remainder}} \end{array} \qquad \begin{array}{r|l} & q \\ \hline d & D \\ & \;\;r \end{array}$$

$473 = 6 * 78 + 5$

$$\boxed{D = d * q + r}$$

$\frac{473}{6} = 78 + \frac{5}{6}$

$$\boxed{\frac{D}{d} = q + \frac{r}{d}}$$

Ex: Divide $4x^4 + 3x^2 - 4$ by $2x^2 + x - 3$

$$\begin{array}{r|l} & 2x^2 - x + 5 \\ \hline 2x^2 + x - 3 & 4x^4 + 0x^3 + 3x^2 + 0x - 4 \\ & -(4x^4 + 2x^3 - 6x^2) \\ \hline & \quad\; -2x^3 + 9x^2 + 0x - 4 \\ & \quad -(-2x^3 - x^2 + 3x) \\ \hline & \qquad\quad 10x^2 - 3x - 4 \\ & \qquad\quad 10x^2 + 5x - 15 \\ \hline & \qquad\qquad\; -8x + 11 \end{array}$$

$\frac{4x^4}{2x^2} = 2x^2$

$\frac{-2x^3}{2x^2} = -x$

$\frac{10x^2}{2x^2} = 5$

$$D = d \cdot q + r$$

$$4x^4 + 3x^2 - 4 = (2x^2 + x - 3)(2x^2 - x + 5) + (-8x + 11)$$

$$\frac{D}{d} = q + \frac{r}{d}$$

$$\frac{4x^4 + 3x^2 - 4}{2x^2 + x - 3} = (2x^2 - x + 5) + \frac{-8x + 11}{2x^2 + x - 3}$$

SYNTHETIC DIVISION

When a polynomial of degree one or higher is divided by $x - a$, we can use a shorter division called synthetic Division

Ex: Divide $3x^3 - 4x + 60$ by $x + 3$

$$x - a = x + 3$$
$$-a = 3$$
$$a = -3$$

$$\begin{array}{r|rrrr}
 & 3 & 0 & -4 & 60 \\
 & & + & + & + \\
a = -3 & \downarrow & -9 & +27 & -69 \\
\hline
 & \underbrace{3 \quad -9 \quad 23}_{\text{quotient}} & & & -9 \leftarrow \text{remainder}
\end{array}$$

$$3x^3 - 4x + 60 = (3x^2 - 9x + 23)(x + 3) - 9$$

$$D = d \cdot q + r$$

ZEROES AND FACTORS OF A POLYNOMIAL

$P(x) = x^3 + 4x^2 + x - 6$

$P(1) = 1 + 4 + 1 - 6 = 0$

$x = 1$ is a zero (or a root) of $P(x)$

Factor theorem: If $P(r) = 0$ then $x - r$ is a factor of $P(x)$ (and viceversa)

So $x-1$ is a factor of x^3+4x^2+x-6

this means that the remainder of the division $\dfrac{x^3+4x^2+x-6}{x-1}$ is zero.

$$\begin{array}{r|rrrr} & 1 & 4 & 1 & -6 \\ 1 & \downarrow & 1 & 5 & 6 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$

$$x^3+4x^2+x-6 = (x-1)(x^2+5x+6)$$

$$D = d \cdot q$$

theorem: If a polynomial has integer roots they must be divisors of the independent term

Ex: $P(x) = x^3+4x^2+x-6$

divisors of -6: $\pm 1, \pm 2, \pm 3, \pm 6$