

POLYNOMIAL FUNCTIONS

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

where  $a_n, a_{n-1}, \dots, a_1, a_0$  are real numbers (called the coefficients) and the exponents of the variable are non-negative integers

Example

$$f(x) = 2x^2 - 3x + \sqrt{2} \quad \leftarrow x^0 \quad \text{polynomial}$$

$$f(y) = 2y^2 - \left(\frac{3}{y}\right) + 4 \quad \text{not a polynomial}$$

$$\frac{3}{y} = 3y^{-1} \quad \leftarrow \text{exponent is } < 0$$

$$f(z) = (-2\sqrt{y}) + 3x - 4 \quad \text{not a polynomial}$$

$$\sqrt{y} = y^{1/2} \quad \leftarrow \text{not an integer}$$

The degree of a polynomial is the largest power of the variable in the polynomial

Example

$$f(x) = 2x^2 + 3$$

degree

2

$$f(x) = 3x$$

1

$$f(x) = 3 = 3x^0$$

0

$$f(x) = 0$$

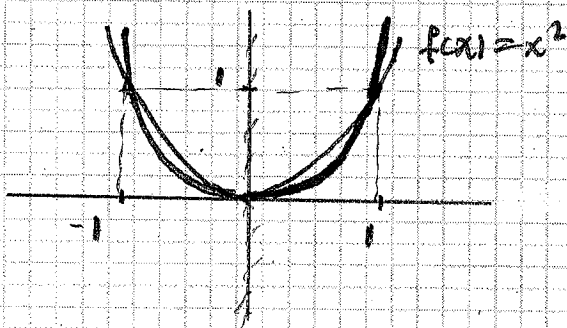
not-defined

$$f(x) = 2x^3 - 3x^2 + x - 2$$

3

Power Functions

$$f(x) = ax^n$$



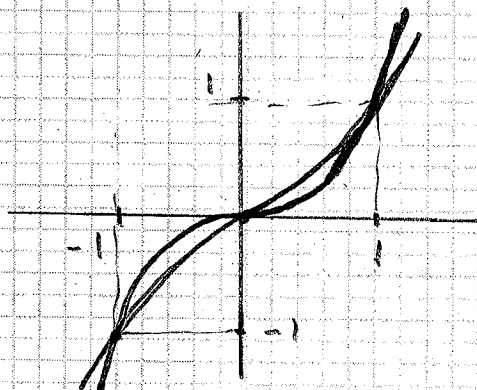
$$f(x) = x^2$$

$$f(x) = x^4$$

EVEN

$$\text{as } x \rightarrow \infty, y \rightarrow \infty$$

$$\text{as } x \rightarrow -\infty, y \rightarrow \infty$$



$$f(x) = x^3$$

$$f(x) = x^5$$

ODD

$$\text{as } x \rightarrow \infty, y \rightarrow \infty$$

$$\text{as } x \rightarrow -\infty, y \rightarrow -\infty$$

Graphing Polynomials Using Transformations

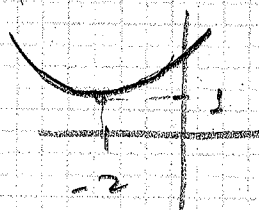
ex: 
$$f(x) = x^2 + 4x + 5$$

$$f(x) = x^2 + 4x + 5 = x^2 + 4x + 4 + 5 - 4$$

$\downarrow$        $\swarrow$  squared  
 $(+2)$

$$f(x) = (x+2)^2 + 1$$

$$f(x) = x^2 \rightarrow f(x) = (x+2)^2 \rightarrow f(x) = (x+2)^2 + 1$$



ZEROS OF A POLYNOMIAL

$$f(x) = x^2 + 5x + 6$$

$$x^2 + 5x + 6 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{25 - 4 \cdot 1 \cdot 6}}{2 \cdot 1} = \frac{-5 \pm 1}{2} = \begin{cases} -6/2 = -3 \\ -4/2 = -2 \end{cases}$$

Δ factoring  $x^2 + 5x + 6 = (x+2)(x+3) = 0$

$$\Rightarrow x+2=0 \quad \text{or} \quad x+3=0$$

$$x = -2$$

mult one

$$x = -3$$

mult one

$$f(x) = (x+2)(x+3)$$

Ex:  $f(x) = x^2(x+3)(x-2)$

$$x=0$$

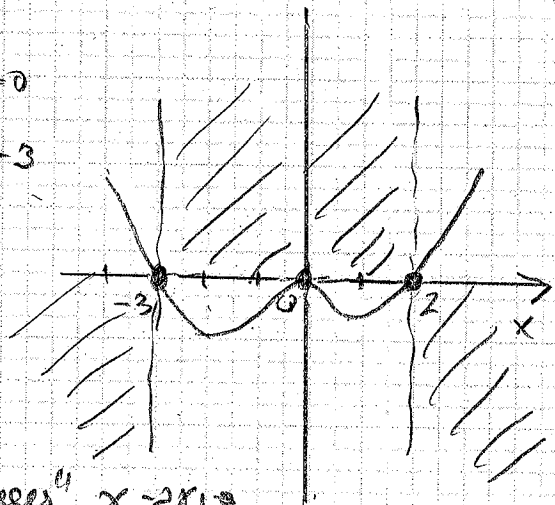
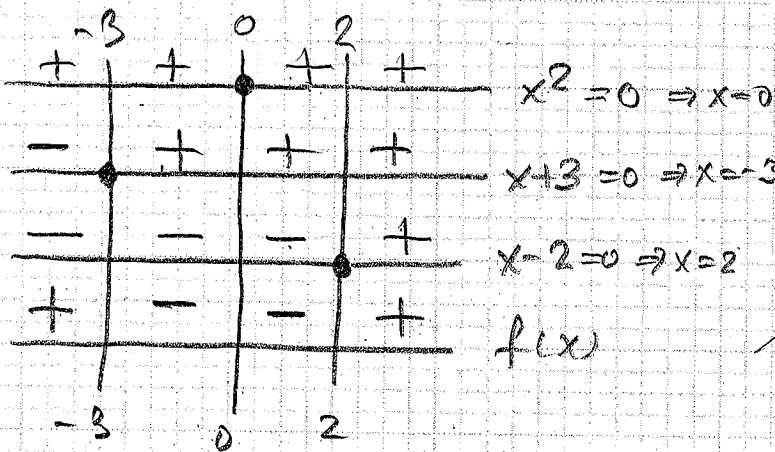
mult 2

$$x=-3$$

mult 1

$$x=2$$

mult 1



multiplicity is odd the graph "crosses" x-axis  
 " " even " " "touches" " "