PRODUCT-TO-SUM AND SUM-TO-PRODUCT FORMULAS

Product-to-sum Formulas

$$\boxed{\begin{aligned} &\sin\alpha \cdot \sin\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right] && (1) \\ &\cos\alpha \cdot \cos\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right] && (2) \\ &\sin\alpha \cdot \cos\beta = \tfrac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right] && (3) \end{aligned}}$$

Proof (1)

$$\cos(\alpha-\beta) = \cancel{\cos\alpha\cos\beta} + \sin\alpha\sin\beta$$

$$- \quad \cos(\alpha+\beta) = \cancel{\cos\alpha\cos\beta} - \sin\alpha\sin\beta$$

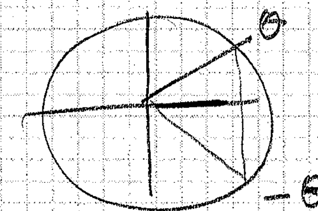
$$\cos(\alpha-\beta) - \cos(\alpha+\beta) = 2\sin\alpha\sin\beta$$

$$\frac{\cos(\alpha-\beta) - \cos(\alpha+\beta)}{2} = \sin\alpha\sin\beta$$

Ex: Express the following product as a sum

$$\sin(\underset{\alpha}{2\theta}) \cdot \sin(\underset{\beta}{3\theta}) = \frac{1}{2}\left[\cos(-\theta) - \cos(5\theta)\right]$$

$$= \frac{1}{2}\left[\cos\theta - \cos(5\theta)\right]$$



SUM-TO-PRODUCT FORMULAS

$$\sin\alpha + \sin\beta = 2 \cdot \sin\frac{\alpha+\beta}{2} \cdot \cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2 \cdot \sin\frac{\alpha-\beta}{2} \cdot \cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha + \cos\beta = 2 \cdot \cos\frac{\alpha+\beta}{2} \cdot \cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2} \cdot \sin\frac{\alpha-\beta}{2}$$

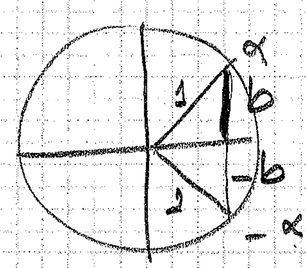
Ex: Express the following sum as a product

$a-b=a+(-b)$

$\sin(3\theta)-\sin(4\theta)=$

$= 2 \cdot \sin\left(-\frac{\theta}{2}\right) \cdot \cos\left(\frac{7\theta}{2}\right) =$

$= -2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{7\theta}{2}\right)$



Prove the identity:

$$\frac{\cos\theta - \cos(3\theta)}{\sin\theta + \sin(3\theta)} = \tan\theta$$

$\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$

Proof

$$\frac{-\cancel{2}\cdot\cancel{\sin(2\theta)}\cdot\sin(-\theta)}{\cancel{2}\,\cancel{\sin(2\theta)}\cdot\cos(-\theta)} = \frac{-(-\sin\theta)}{\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$