PROPERTIES OF LOGARITHMS

1) $\log_b 1 = 0$ because $b^0 = 1$

2) $\log_b b = 1$ " $b^1 = b$

3) $b^{\log_b x} = x$ $\log_2 8 =$

Proof

$\log_b x = n \Rightarrow b^n = x \Rightarrow b^{\log_b x} = x$

4) $\log_b b^x = x$ because $b^x = b^x$

5) $\log_b (x \cdot y) = \log_b x + \log_b y$

Proof:

$\log_b x = M \Rightarrow x = b^M$

$\log_b y = N \Rightarrow y = b^N$

$\log_b (x \cdot y) = \log_b (b^M \cdot b^N) = \log_b b^{M+N} = M + N$

$= \log_b x + \log_b y$

6) $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

7) $\log_b x^n = n \cdot \log_b x$

8) $M = N \Rightarrow \log_b M = \log_b N$

9) $\log_b M = \log_b N \Rightarrow M = N$

THE CHANGE OF BASE FORMULA

$\log_2 8 = 3$

$$\log_2 8 = \frac{\log_{10} 8}{\log_{10} 2} = \frac{\log 8}{\log 2} = 3$$

or

$$\log_2 8 = \frac{\log_e 8}{\log_e 2} = \frac{\ln 8}{\ln 2} = 3$$

$\log_2 8 = 3$

PROBLEMS

write each log as a sum and/or difference of logs.
write powers as factors

1) $\log_3(27x) = \log_3 27 + \log_3 x = \log_3 3^3 + \log_3 x$

$= 3 \cdot \log_3 3 + \log_3 x = 3 + \log_3 x$

2) $\log\left[\frac{x^4\sqrt[3]{(x+1)^1}}{x^2-1}\right] = \log\left[\frac{x^4(x+1)^{1/3}}{(x+1)(x-1)}\right]$

$\sqrt[n]{x^m} = x^{m/n}$

$= \log\left[x^4(x+1)^{1/3}\right] - \log\left[(x+1)(x-1)\right]$

$= \log x^4 + \log(x+1)^{1/3} - \left(\log(x+1) + \log(x-1)\right)$

$= 4\log x + \frac{1}{3}\log(x+1) - \log(x+1) - \log(x-1)$

$= 4\log x + \frac{1}{3}\left[\log x + \log 1\right] - \left[\log x + \log 1\right] - \left[\log x - \log 1\right]$

$= 4\log x - \frac{2}{3}\log(x+1) - \log(x-1)$

Write each expression as a single log

3) $5\log_3(2x+4) - 2\log_3(3x-2) - \log_3 x =$

$= \log_3(2x+4)^5 - \log_3(3x-2)^2 - \log_3 x$

$= \log_3\left[\frac{(2x+4)^5}{(3x-2)^2}\right] - \log_3 x$

$= \log_3\left[\frac{\frac{(2x+4)^5}{(3x-2)^2}}{x}\right] = \log_3\left[\frac{(2x+4)^5}{(3x-2)^2} \cdot \frac{1}{x}\right]$

$ab = ba$

$= \log_3\left[\frac{(2x+4)^5}{x(3x-2)^2}\right]$