

QUADRATIC EQUATIONS WITH COMPLEX SOLUTIONS

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b^2 - 4ac = \text{discriminant}$$

$$b^2 - 4ac > 0 \quad \text{two distinct real solutions}$$

$$b^2 - 4ac = 0 \quad \text{one real solution (repeated)}$$

$$b^2 - 4ac < 0 \quad \text{two complex solutions} \\ \text{(conjugates of each other)}$$

Example:

$$x^2 + x + 1 = 0$$

$$a=1 \quad b=1 \quad c=1$$

$$x = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3} \cdot \sqrt{-1}}{2}$$

$$= \frac{-1 \pm \sqrt{3}i}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i = \left. \begin{array}{l} -\frac{1}{2} - \frac{\sqrt{3}}{2}i \\ -\frac{1}{2} + \frac{\sqrt{3}}{2}i \end{array} \right\}$$

Example: without solving the equations, determine whether they have two distinct real solutions, one repeated real solution or two complex solutions

$$1) \quad x^2 + 5x + 6 = 0 \quad \begin{array}{l} \underline{b^2 - 4ac} \\ 25 - 4 \cdot 1 \cdot 6 = 25 - 24 > 0 \quad \text{two real sol} \end{array}$$

$$2) \quad x^2 + 4x + 4 = 0 \quad 16 - 4 \cdot 1 \cdot 4 = 0 \quad \text{one real sol}$$

$$3) \quad x^2 + 2x + 4 = 0 \quad 4 - 4 \cdot 1 \cdot 4 = -12 < 0 \quad \text{two complex sol.}$$