QUADRATIC EQUATIONS

$$ax^2+bx+c=0$$

Ex: $2x^2+5x+3=0$

$a=2 \quad b=5 \quad c=3$

1) Quadratic Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-5\pm\sqrt{25-4\cdot 2\cdot 3}}{2\cdot 2}$$

$$=\frac{-5\pm 1}{4}=\begin{cases}\frac{-6}{4}=-3/2\\ \frac{-4}{4}=-1\end{cases}$$

$b^2-4ac>0$ two solutions

$b^2-4ac=0$ one solution

$b^2-4ac<0$ no real solution

2) Factoring

$2x^2+5x+3=0$ $\quad a=2 \; b=5 \; c=3$

$\cdot = a\cdot c = 6 \quad 2,3$

$+ = b = 5$

$$2x^2+5x+3 = 2x^2+2x+3x+3$$

$$= 2x(x+1)+3(x+1)$$

$$= (x+1)(2x+3) = 0$$

$$\boxed{a \cdot b = 0 \Rightarrow a=0 \text{ or } b=0}$$

$x+1=0$ or $2x+3=0$

$x=-1$ $\quad 2x=-3$

$x=-3/2$

3) Square Root Method

$$x^2 = a$$

$x^2-a=0$ $\qquad x^2-a^2=(x+a)(x-a)$

$(x+\sqrt{a})(x-\sqrt{a})=0$

$x+\sqrt{a}=0$ or $x-\sqrt{a}=0$

$x=-\sqrt{a}$ or $x=\sqrt{a}$

Ex: $(x-3)^2=4$

$x-3=\sqrt{4}$ or $x-3=-\sqrt{4}$

$x-3=2$ $\quad x-3=-2$

$x=5$ $\quad x=1$

4) Completing the square

Perfect Square

$$(x+a)^2 = x^2 + 2ax + a^2$$

$2ax$: a

a^2: a

Ex: $x^2 + 4x + 4 = 0$

$4x$: 2

$$(x+2)^2 = 0$$

$$x+2 = \sqrt{0} \quad \text{or} \quad x+2 = -\sqrt{0}$$

$$x+2 = 0 \qquad x+2 = 0$$

$$\boxed{x = -2}$$

Ex: $2x^2 + 5x + 3 = 0$

$$2x^2 + 5x = -3$$

$$x^2 + \frac{5}{2}x + \frac{25}{16} = -3/2 + \frac{25}{16}$$

$\frac{5}{2}x$: $+5/4$

$$\left(x + \frac{5}{4}\right)^2 = \frac{1}{16}$$

$$x + \frac{5}{4} = \sqrt{\frac{1}{16}} \quad \text{or} \quad x + \frac{5}{4} = -\sqrt{\frac{1}{16}}$$

$$x + \frac{5}{4} = \frac{1}{4} \quad \text{or} \quad x + \frac{5}{4} = -\frac{1}{4}$$

$$x = -\frac{4}{4} = -1 \qquad x = -\frac{6}{4} = -3/2$$