

Quadratic Functions, Part II

$$y = a(x-h)^2 + k$$

Ex 1: $y = x^2 - 2x - 3$
 $a = 1 > 0 \checkmark$

$$y = (x^2 - 2x + 1) - 3 - 1$$

half \downarrow \uparrow squared

$$y = (x-1)^2 - 4$$

$$h = 1 \quad k = -4$$

y-int

$$x = 0 \Rightarrow y = x^2 - 2x - 3$$

$$y = -3$$

x-int

$$y = 0 \Rightarrow y = x^2 - 2x - 3$$

$$x^2 - 2x - 3 = 0$$

$$\begin{array}{r|l} \bullet = -3 & -3, -1 \\ + = -2 & \end{array}$$

$$y = (x-3)(x+1) = 0$$

$$x-3=0 \Rightarrow x=3$$

or

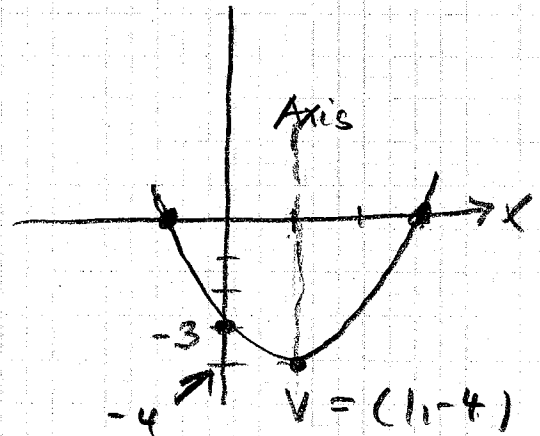
$$x+1=0 \Rightarrow x=-1$$

Completing
the square

$$(x+a)^2 = x^2 + 2ax + a^2$$

\downarrow
 a

$$(x-a)^2 = x^2 - 2ax + a^2$$



$$\text{Dom} = (-\infty, \infty)$$

$$\text{Range} = [-4, \infty)$$

$$\text{Axis: } x = 1$$

Ex 2: $y = -x^2 + 2x + 3$

$$a = -1 < 0 \quad \curvearrowright$$

$$y = -(x^2 - 2x + 1) + 3 + 1$$

$$y = -(x - 1)^2 + 4$$

$$h = 1 \quad k = 4$$

y-int

$$x = 0 \Rightarrow y = -x^2 + 2x + 3$$

$$y = 3$$

x-int

$$y = 0 \Rightarrow y = -x^2 + 2x + 3 = 0$$

$$x^2 - 2x - 3 = 0$$

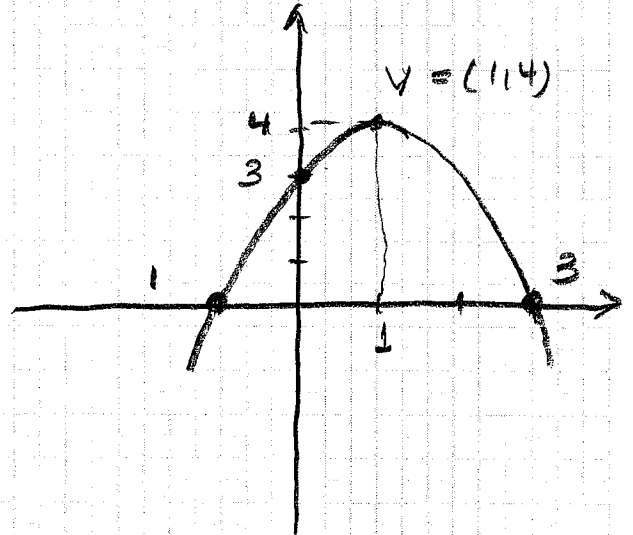
$$\begin{array}{r|l} 0 & = -3 \quad | \quad -3, 1 \\ + & = -2 \end{array}$$

$$(x - 3)(x + 1) = 0$$

$$x - 3 = 0 \Rightarrow x = 3$$

or

$$x + 1 = 0 \Rightarrow x = -1$$



Ex 3: $y = -3x^2 + 3x + 6$

$$y = -3 \left(x^2 - x + \frac{1}{4} \right) + 6 + \frac{3}{4}$$

\downarrow
 $-\frac{1}{2}$

$$y = -3 \left(x - \frac{1}{2} \right)^2 + \frac{27}{4}$$

$$h = \frac{1}{2} \quad k = \frac{27}{4} = 6.75$$

y-int

$$x=0 \Rightarrow y = -3x^2 + 3x + 6$$

$$y = 6$$

x-int

$$y=0 \Rightarrow y = -3x^2 + 3x + 6$$

$$-3x^2 + 3x + 6 = 0$$

$$x^2 - x - 2 = 0$$

$$\begin{array}{r|l} 0 & = -2 & | & -2, 1 \\ + & = -1 & & \end{array}$$

$$(x-2)(x+1) = 0$$

$$x-2=0 \Rightarrow x=2$$

or

$$x+1=0 \Rightarrow x=-1$$

$$y = -3x^2 + 4x + 6$$

$$-3 \left(x^2 - \frac{4}{3}x \right) + 6$$

