RADICAL EQUATIONS

$\sqrt{x+2} - 3 = 4$

The square root of x is a number y such that $y^2 = x$

For example, 2 is a square root of 4 because $2^2 = 4$

-2 is also a square root of 4 because $(-2)^2 = 4$

Every non-negative number x has two square roots that we denote by $\sqrt{x}$ and $-\sqrt{x}$

The principal square root of x is $\sqrt{x}$ and it is a non-negative number

For example

The principal square root of 25 is $\sqrt{25} = 5$. The other root of 25 is $-\sqrt{25} = -5$

So, you can say that 25 has two square roots: ± 5 or you can write that the roots are $\sqrt{25}$ and $-\sqrt{25}$

But, don't write that $\sqrt{25} = \pm 5$

The symbol $\sqrt{25}$ is reserved for the principal square root of 25 which is a non-negative number: 5

NOTE: $\sqrt{x^2} = |x|$

Ex: $\sqrt{4^2} = |4| = 4$

$\sqrt{(-4)^2} = |-4| = 4$

Solve: $\sqrt{x+2} - 3 = 4$

$\sqrt{x+2} = 7$

$(\sqrt{x+2})^2 = 7^2$

$|x+2| = 49$

$x+2 = 49$

$x = 47$

Domain
$x+2 \geq 0$

check
$\sqrt{47+2} - 3 = 4$

$\sqrt{49} - 3 = 4$

$7 - 3 = 4$ ✓

Ex: $x = -6$

$x^2 = 36$

$x^2 - 36 = 0$

$(x+6)(x-6) = 0$

$x+6 = 0$ or $x-6 = 0$

$x = -6$ $x = 6$

Example: Solve

$\sqrt{x+2} - \sqrt{x+3} = 4$

$(a-b)^2 = a^2 - 2ab + b^2$

$\left(\underbrace{\sqrt{x+2}}_{a} - \underbrace{\sqrt{x+3}}_{b}\right)^2 = 4^2$

$(\sqrt{x+2})^2 - 2\sqrt{x+2}\sqrt{x+3} + (\sqrt{x+3})^2 = 16$

$|x+2| - 2\sqrt{(x+2)(x+3)} + |x+3| = 16$

$x+2 - 2\sqrt{(x+2)(x+3)} + x+3 = 16$

$2\sqrt{(x+2)(x+3)} = 11 - 2x$

$(2\sqrt{(x+2)(x+3)})^2 = (11-2x)^2$

$4(x+2)(x+3) = (11-2x)^2$

$4(x^2+5x+6) = 121 - 44x + 4x^2$

$4x^2 + 20x + 24 = 121 - 44x + 4x^2$

$64x = 97$

$x = \frac{97}{64}$