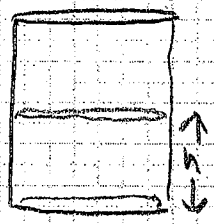
RELATED RATES



r = constant
h = variable
volume V = variable

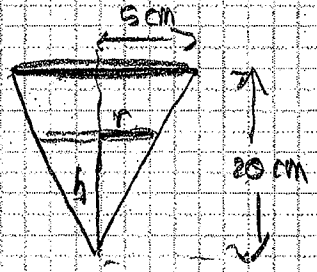
$V = \pi r^2 h$

$\frac{dh}{dt}$ = given

$\pi r^2 h$

$\frac{dV}{dt} = ?$

$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$

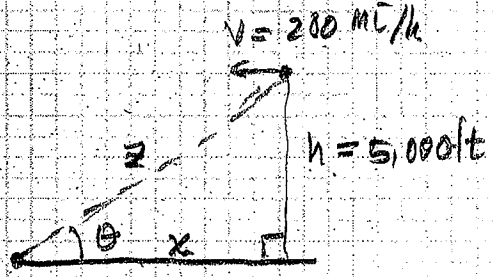


r = variable
h = variable

$\frac{dV}{dt} = 2 \frac{cm^3}{sec}$

$V = \frac{1}{3} \pi r^2 h$

$\frac{dh}{dt} = ?$



h = constant
θ = variable.

$\frac{dx}{dt}$ = given = 280 mi/h

$\tan\theta = \frac{h}{x}$

$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}(h \cdot x^{-1})$

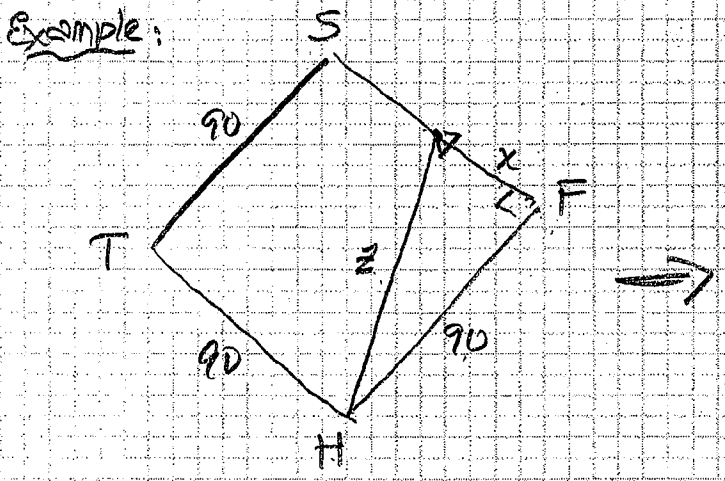
$\sec^2\theta \cdot \frac{d\theta}{dt} = h(-1)x^{-2}\frac{dx}{dt}$

steps

1) Draw the situation
2) choose names for the variables
3) Translate the information in the problem about rates into derivatives
4) We need a formula that relates the variables and constants in the problem
5) Derive implicitly the given formula

Typically, the formulas needed will be:
volumes, areas, Pythagorean Theorem, Trig.

Example:



$\Rightarrow z^2 = 90^2 + 30^2$

$z^2 = 9000$

$z \approx 98.9$ ft

speed of runner $= 28 \frac{ft}{s} = \frac{dx}{dt}$

How fast is z changing? $= \frac{dz}{dt} = ?$ when $x = 30$ ft

$z^2 = 90^2 + x^2 \Rightarrow \frac{d}{dt}(z^2) = \frac{d}{dt}(90^2 + x^2)$

$\Rightarrow 2z \cdot \frac{dz}{dt} = 2x \cdot \frac{dx}{dt} \Rightarrow 2 * 98.9 \frac{dz}{dt} = 2 * 30 * 28$

$\Rightarrow 197.8 \frac{dz}{dt} = 1680 \Rightarrow \frac{dz}{dt} = \frac{1680}{197.8} \approx 8.5 \frac{ft}{s}$