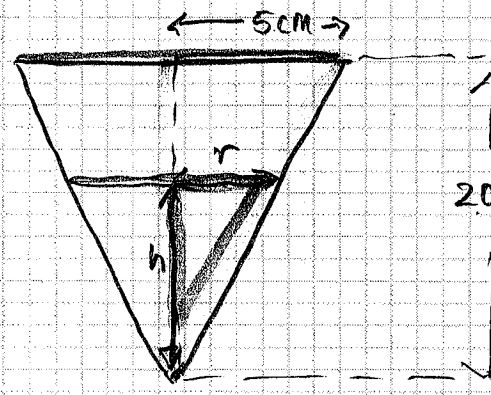
RELATED RATES, PART II



$\frac{dV}{dt} = -\frac{3\,cm^3}{min}$ "losing"

Question: How fast is the height of the liquid changing?

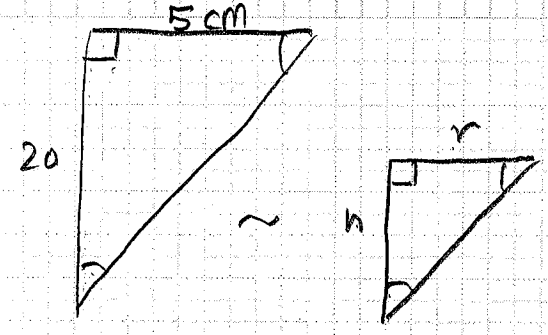
$\frac{dh}{dt} = ?$ (when $h = 11$ cm)

$V = \frac{1}{3}\pi r^2 h$ ← product rule

$\frac{d}{dt}(r^2) = 2r\frac{dr}{dt}$

$\frac{dV}{dt} = \frac{1}{3}\pi\left[2r\cdot\frac{dr}{dt}\cdot h + r^2\cdot\frac{dh}{dt}\right]$

Can we write r as a function of h?



similar triangles have their corresponding sides proportional

$\frac{r}{h} = \frac{5}{20} \Rightarrow r = \frac{5h}{20} = \frac{h}{4}$

$V = \frac{\pi}{3}\left(\frac{h}{4}\right)^2\cdot h \Rightarrow V = \frac{\pi}{3}\cdot\frac{h^3}{16} \Rightarrow \boxed{V = \frac{\pi}{48}h^3}$

$\frac{dV}{dt} = \frac{\pi}{48}3h^2\cdot\frac{dh}{dt} \Rightarrow -3 = \frac{\pi}{16}\cdot 11^2\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = -\frac{3*16}{121\pi}$

$\frac{dh}{dt} = -\frac{48}{121\pi}$