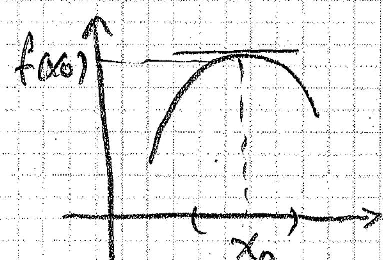
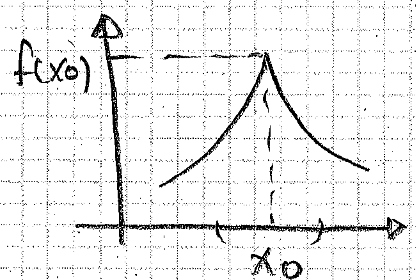
RELATIVE MAXIMUM AND MINIMUM

One extremum refers to one maximum or minimum

If there is more than one extremum, we talk about extrema. For example: two extrema

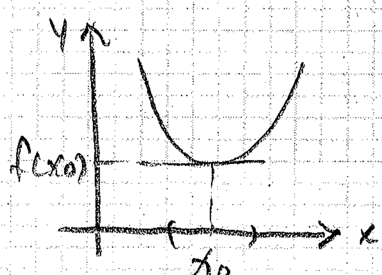
Relative Maximum at $x = x_0$ if $f(x_0) \geq f(x)$

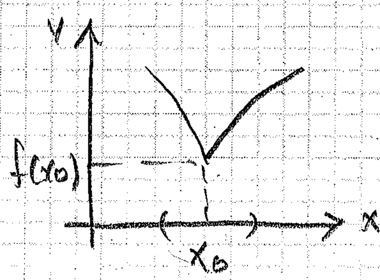




in an open interval containing x_0

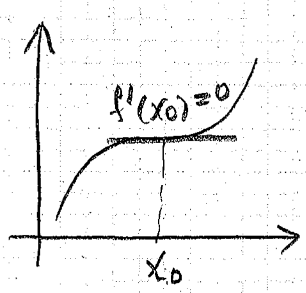
Relative Minimum at $x = x_0$ if $f(x_0) \leq f(x)$

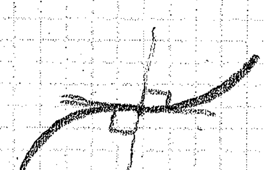


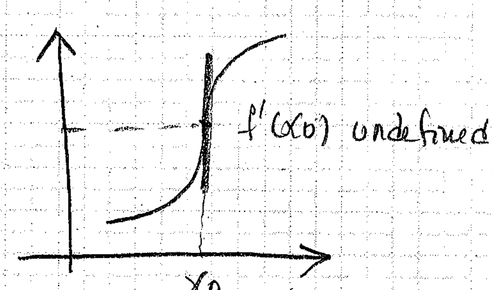


in an open interval containing x_0

A critical Point for a function f is a value of x, in the domain of f where $f'(x) = 0$ or $f'(x)$ is undefined.

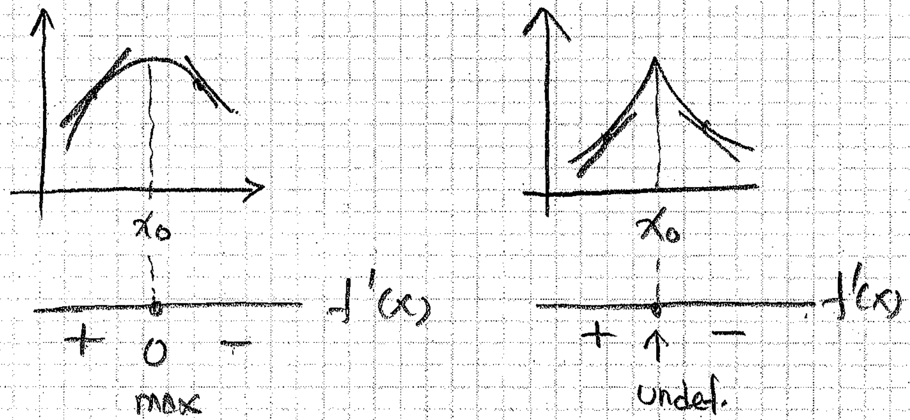




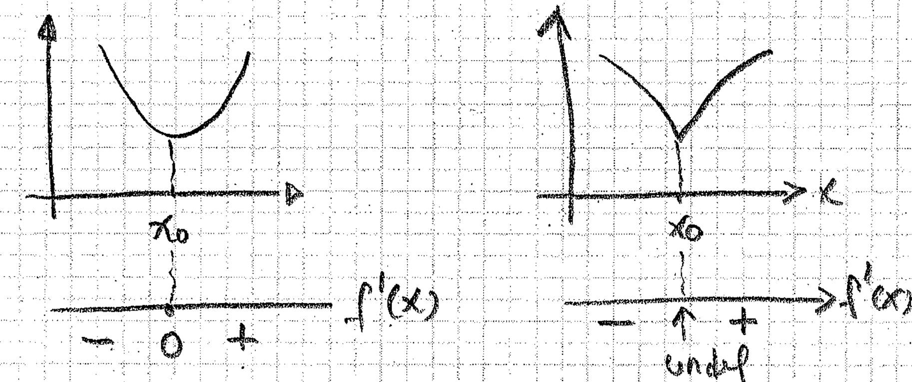


THE FIRST DERIVATIVE TEST

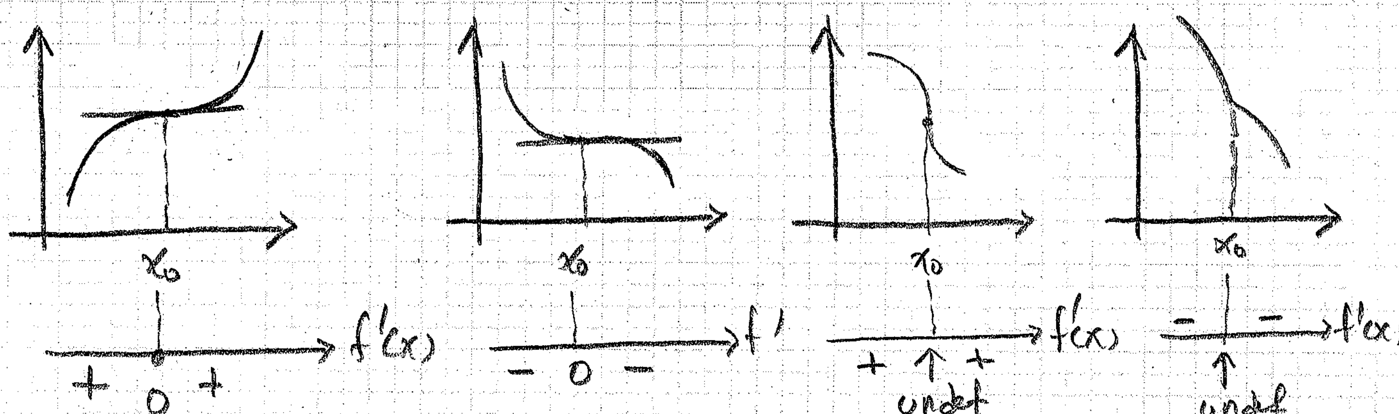
Let f be continuous at the critical point x_0



Maximum



Minimum



Example: Find the relative extrema of the function

$f(x) = x^2 - 4x + 3$

Domain $= \mathbb{R} = (-\infty, \infty)$

Critical Points

$f'(x) = 2x - 4$

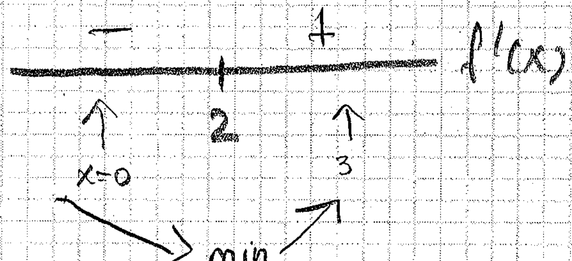
$2x - 4 = 0$

$2x = 4$

$\boxed{x = 2}$ C.P.

$2x - 4 =$ undef

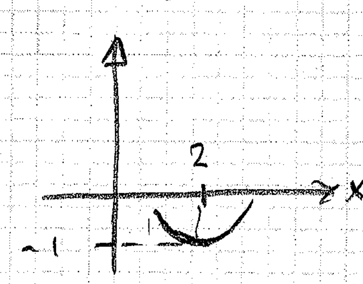
No C.P. over here



At $x = 2$, $f(x)$ has a relative minimum

$x_{min} = 2$

$y_{min} = 2^2 - 4 \cdot 2 + 3 = -1$



Example Find the relative extrema of the function

$$f(x) = \sqrt[3]{x-2} + 1 = (x-2)^{1/3} + 1$$

Domain $= (-\infty, \infty) = \mathbb{R}$

Critical Points

$$f'(x) = \frac{1}{3}(x-2)^{-2/3} = \frac{1}{3\sqrt[3]{(x-2)^2}}$$

$\frac{1}{3\sqrt[3]{(x-2)^2}} = 0$ no solution

no C.P. occur here

$\frac{a}{b} = 0 \Rightarrow a = 0$ and $b \neq 0$

$\frac{0}{4} = 0$ $\quad \frac{0}{0} = 2$ $\quad \frac{4}{0} =$ undf

$\frac{1}{3\sqrt[3]{(x-2)^2}}$ undef $\Rightarrow$

$\Rightarrow x-2 = 0 \Rightarrow \boxed{x=2}$

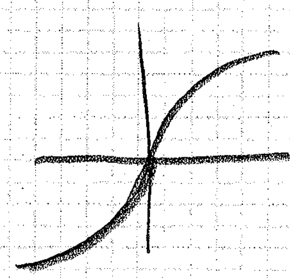
C.P

$+ \quad + \quad f'(x)$

$x=0 \quad 2 \quad x=3$

no max or min

$f(x) = \sqrt[3]{x}$



$f(x) = \sqrt[3]{x-2} + 1$