ROOTS

The square root of 4 is a number that squared gives you 4

Therefore

2 is a square root of 4 because $2^2 = 4$

-2 " " " $(-2)^2 = 4$

4 has two square roots that we denote by

$\sqrt{4}$ and $-\sqrt{4}$

The principal square root of x is $\sqrt{x}$ and it is a non-negative number

You can say that 4 has two square roots:

$\sqrt{4} = 2$ and $-\sqrt{4} = -2$ but don't write $\sqrt{4} = \pm 2$ $\sqrt{4}$ is 2 (not ± 2)

Example: $\sqrt{3^2} = 3$

$\sqrt{(-3)^2} = \sqrt{9} = 3$

~~$\sqrt{(-3)^2} = -3$~~ wrong.

NOTE

$\sqrt{x^2} = |x|$

$\sqrt{(-3)^2} = |-3| = 3$

Note: negative numbers don't have real square roots

$\sqrt{-9}$

$3 \cdot 3 = 9$

$(-3)(-3) = 9$

Everything we have said about square roots can be extended to even-indexed roots

Example: the principal fourth root of x is $\sqrt[4]{x}$ and it is a non-negative number

$\sqrt[4]{16} = 2$ because $2^4 = 16$

$\sqrt[4]{x^4} = |x|$

Negative numbers don't have (real) even-indexed roots

$\sqrt[4]{-16}$ $2^4 = 16$
$(-2)^4 = 16$

what about odd-indexed numbers?

$\sqrt[3]{27} = 3$ $3^3 = 3 \cdot 3 \cdot 3 = 27$

$\sqrt[3]{-27} = -3$ $(-3)^3 = (-3)(-3)(-3) = 9(-3) = -27$

Negative numbers do have real odd-indexed roots

$\sqrt[n]{x}$ is called a radical

$n \geq 2$ is the index

x is the radicand

Properties of Radicals

1) $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

2) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

3) $\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

4) $\sqrt[n]{a^m} = a^{m/n}$

Exercises: Simplify

1) $\sqrt{2^6} = 2^{6/2} = 2^3 = 8$

$2^{7/2} = 2^{3.5}$

2) $\sqrt{2^7} = \sqrt{2^6 \cdot 2} = \sqrt{2^6} \cdot \sqrt{2} = 2^3\sqrt{2}$

$2 \overline{)\,7}$ — 3, 6, 1

3) $\sqrt[3]{-81x^7} = \sqrt[3]{(-1)3^4 x^7} = \sqrt[3]{(-1)}\sqrt[3]{3^4}\sqrt[3]{x^7}$

$-81 = (-1)81 = (-1)3^4$

$= (-1)3\sqrt[3]{3} \cdot x^2\sqrt[3]{x}$

$= -3x^2\sqrt[3]{3x}$

81	3
27	3
9	3
3	3
1	

$3 \overline{)\,4}$ — 1, 3, 1 → exp out; 1 → exp in

$3 \overline{)\,7}$ — 2, 6, 1 → out; 1 → in

4) $\sqrt[5]{\frac{64x^7}{y16}} = \sqrt[5]{\frac{2^6x^7}{y16}} = \frac{2x}{y^3}\sqrt[5]{\frac{2x^2}{y}}$

64	2
32	2
16	2
8	2
4	2
2	2
1	

$5\overline{)6}$ → 1 out, 5, 1 in

$5\overline{)7}$ → 1 out, 5, 2 in

$5\overline{)16}$ → 3 out, 15, 1