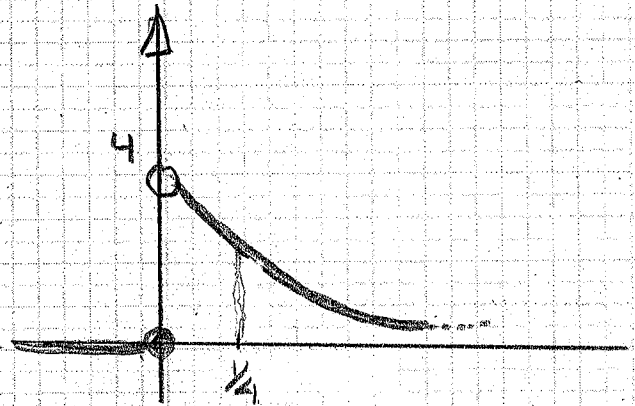


EXPECTED VALUE EXAMPLE (For Engineers)

$$f(x) = \begin{cases} 4e^{-4x} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$



$$E(x) = \int_{-\infty}^{\infty} x f(x) dx = \int_0^{\infty} x 4e^{-4x} dx$$

Improper Integral

$$= \lim_{t \rightarrow \infty} \int_0^t 4x e^{-4x} dx$$

$$I_1 = \int 4x e^{-4x} dx = 4 \int x e^{-4x} dx = uv - \int v du$$

By parts

$$u = x \quad dv = e^{-4x} dx$$

$$du = dx \quad v = \frac{e^{-4x}}{-4}$$

$$\int e^{kx} dx = \frac{e^{kx}}{k} + C$$

$$I_1 = 4 \left[\frac{x e^{-4x}}{4} - \int \frac{e^{-4x}}{-4} dx \right] = -x e^{-4x} + \int e^{-4x} dx$$

$$-x e^{-4x} + \frac{e^{-4x}}{-4} + C = -x e^{-4x} - \frac{1}{4} e^{-4x} + C$$

$$= -\frac{x}{e^{4x}} - \frac{1}{4 e^{4x}} + C$$

$$I_2 = \int_0^t 4x e^{-4x} dx = -\frac{x}{e^{4x}} - \frac{1}{4e^{4x}} \Big|_0^t$$

$$= -\frac{t}{e^{4t}} - \frac{1}{4e^{4t}} - \left(-\frac{0}{e^0} - \frac{1}{4e^0} \right) \quad e^0 = 1$$

$$= -\frac{t}{e^{4t}} - \frac{1}{4e^{4t}} + \frac{1}{4}$$

$$E(X) = \lim_{t \rightarrow \infty} \left(-\frac{t}{e^{4t}} - \frac{1}{4e^{4t}} + \frac{1}{4} \right) = \frac{1}{4}$$

$$\lim_{t \rightarrow \infty} \frac{-t}{e^{4t}} \stackrel{\text{L'H}}{=} \lim_{t \rightarrow \infty} \left(\frac{-1}{e^{4t} \cdot 4} \right) = 0$$