

THE VARIANCE AND STANDARD DEVIATION for discrete and continuous variables

Discrete

$$\text{Var}(X) = E[(X - E(X))^2]$$

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$\text{SD}(X) = \sqrt{\text{Var}(X)}$$

Example

X	0	1	2	3
$P(X)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$$E(X) = 0 + \frac{1}{4} + 1 + \frac{1}{4} + 2 + \frac{1}{4} + 3 + \frac{1}{4} = \frac{3}{2}$$

$$X - E(X) = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$$

$$[X - E(X)]^2 = \frac{9}{4}, \frac{1}{4}, \frac{1}{4}, \frac{9}{4}$$

$$E[(X - E(X))^2] = \frac{9}{4} * \frac{1}{4} + \frac{1}{4} * \frac{1}{4} + \frac{1}{4} * \frac{1}{4} + \frac{9}{4} * \frac{1}{4} = \frac{9+1+1+9}{16} = \frac{20}{16} = \frac{5}{4}$$

Continuous

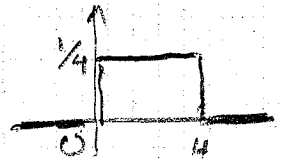
$$\text{Var}(X) = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$$

where $\mu = E(X)$

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

Example $f(x) = \begin{cases} \frac{1}{4} & \text{for } 0 \leq x \leq 4 \\ 0 & \text{elsewhere} \end{cases}$

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx$$



$$E(X) = \int_0^4 x \cdot \frac{1}{4} dx = \frac{1}{4} \int_0^4 x dx = \frac{1}{4} \left. \frac{x^2}{2} \right|_0^4 = \frac{1}{8} (16 - 0) = 2 = \mu$$

$$\text{Var}(X) = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$$

$$= \int_0^4 (x - 2)^2 \frac{1}{4} dx = \frac{1}{4} \int_0^4 (x^2 - 4x + 4) dx = \frac{1}{4} \left(\frac{x^3}{3} - 4 \frac{x^2}{2} + 4x \right) \Big|_0^4 = \frac{4}{3}$$

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$\begin{array}{l} x^2 \quad 0 \quad 1 \quad 4 \quad 9 \\ p(x^2) \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \end{array}$$

$$\begin{aligned} E(X^2) &= 0 + \frac{1}{4} + \frac{4}{4} + \frac{9}{4} \\ &= \frac{14}{4} = \frac{7}{2} \end{aligned}$$

$$\text{Var}(X) = \frac{7}{2} - \left(\frac{3}{2}\right)^2 = \frac{5}{4}$$

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$E(X) = 2$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$$

$$= \int_0^4 x^2 \cdot \frac{1}{4} dx = \frac{1}{4} \int_0^4 x^2 dx$$

$$\begin{aligned} &= \frac{1}{4} \left. \frac{x^3}{3} \right|_0^4 = \frac{1}{12} (4^3 - 0) = \frac{4^3}{12} \\ &= \frac{16}{3} \end{aligned}$$

$$\text{Var}(X) = \frac{16}{3} - 2^2 = \frac{10}{3}$$