Sampling Distributions. Biased and Unbiased Estimators

x	0	6	9
$p(x)$	1/4	1/4	1/2

μ

A random sample of size $n=3$ is taken from the population. Find the sampling distribution of the sample mean, $\bar{x}$, and the sampling distribution of the sample median, M.

Possible samples	$\bar{x}$	M	Probability
0 0 0	0	0	(1/4)(1/4)(1/4) = 1/64
0 0 6	2	0	(1/4)(1/4)(1/4) = 1/64
0 0 9	3	0	(1/4)(1/4)(1/2) = 1/32 = 2/64
0 6 0	2	0	(1/4)(1/4)(1/4) = 1/64
0 6 6	4	6	(1/4)(1/4)(1/4) = 1/64
0 6 9	5	6	(1/4)(1/4)(1/2) = 1/32 = 2/64
0 9 0	3	0	(1/4)(1/2)(1/4) = 1/32 = 2/64
0 9 6	5	6	(1/4)(1/2)(1/4) = 1/32 = 2/64
0 9 9	6	9	(1/4)(1/2)(1/2) = 1/16 = 4/64
6 0 0	2	0	(1/4)(1/4)(1/4) = 1/64
6 0 6	4	6	(1/4)(1/4)(1/4) = 1/64
6 0 9	5	6	(1/4)(1/4)(1/2) = 1/32 = 264
6 6 0	4	6	(1/4)(1/4)(1/4) = 1/64
6 6 6	6	6	(1/4)(1/4)(1/4) = 1/64
6 6 9	7	6	(1/4)(1/4)(1/2) = 1/32 = 2/64
6 9 0	5	6	(1/2)(1/4)(1/4) = 1/32 = 2/64
6 9 6	7	6	(1/2)(1/4)(1/4) = 1/32 = 2/64
6 9 9	8	9	(1/2)(1/4)(1/2) = 1/16 = 4/64
9 0 0	3	0	(1/2)(1/4)(1/4) = 1/32 = 2/64
9 0 6	5	6	(1/2)(1/4)(1/4) = 1/32 = 2/64
9 0 9	6	9	(1/2)(1/4)(1/2) = 1/16 = 4/64
9 6 0	5	6	(1/2)(1/4)(1/4) = 1/32 = 2/64
9 6 6	7	6	(1/2)(1/4)(1/4) = 1/32 = 2/64
9 6 9	8	9	(1/2)(1/4)(1/2) = 1/16 = 4/64
9 9 0	6	9	(1/2)(1/2)(1/4) = 1/16 = 4/64
9 9 6	8	9	(1/2)(1/2)(1/4) = 1/16 = 4/64
9 9 9	9	9	(1/2)(1/2)(1/2) = 1/8 = 8/64

$\bar{x}$	0	2	3	4	5	6	7	8	9
$P(\bar{x})$	$\frac{1}{64}$	$\frac{3}{64}$	$\frac{6}{64}$	$\frac{3}{64}$	$\frac{12}{64}$	$\frac{13}{64}$	$\frac{6}{64}$	$\frac{12}{64}$	$\frac{8}{64}$

M	0	6	9
P(M)	$\frac{10}{64}$	$\frac{24}{64}$	$\frac{30}{64}$

Biased and unbiased Estimators

$$\mu = \sum [x\, p(x)] = 0 * \frac{1}{4} + 6 * \frac{1}{4} + 9 * \frac{1}{2} = 6$$

$$\mu_{\bar{x}} = \sum [\bar{x} \cdot p(\bar{x})] =$$

$$= 0 * \frac{1}{64} + 2 * \frac{3}{64} + 3 * \frac{6}{64} + 4 * \frac{3}{64} + 5 * \frac{12}{64} + 6 * \frac{13}{64} + 7 * \frac{6}{64} + 8 * \frac{12}{64} + 9 * \frac{8}{64}$$

$$= 6$$

$\mu_{\bar{x}} = \mu_x \Rightarrow \bar{x}$ is an <u>un</u>biased estimator of μ

$$\mu_M = 0 * \frac{10}{64} + 6 * \frac{24}{64} + 9 * \frac{30}{64} = \frac{207}{32} \neq \mu = 6$$

$\mu_M \neq \mu \Rightarrow$ the Median, M, is a <u>biased</u> estimator of μ