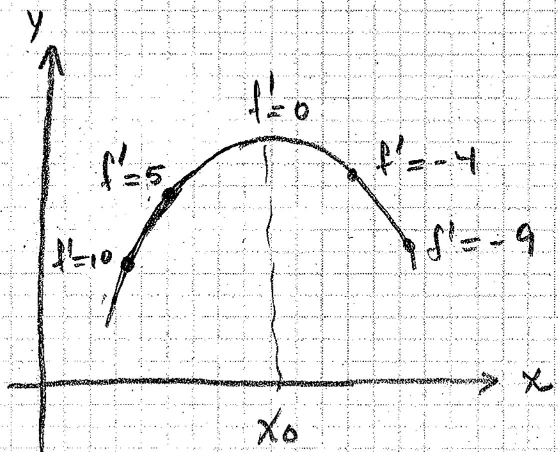
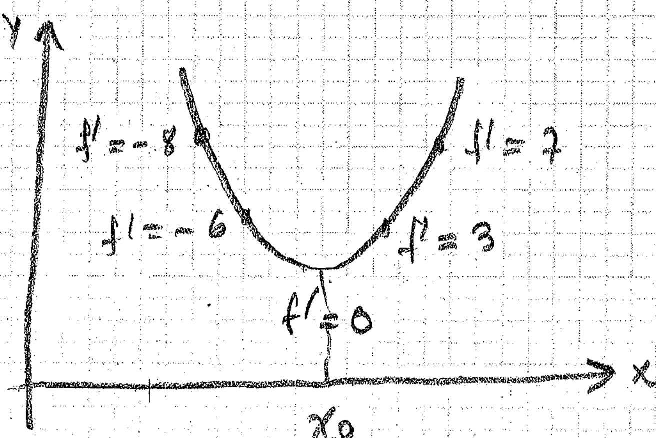
SECOND DERIVATIVE TEST (For relative max & min)



Let f be twice differentiable at x_0

f' is decreasing

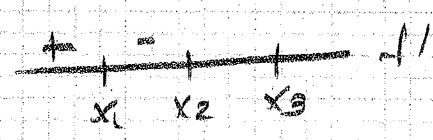
$f''(x_0) < 0 \implies f$ has a local max at x_0



f' is increasing

$f''(x_0) > 0 \implies f$ has a local min at x_0

If $f''(x_0) = 0 \implies$ the test is inconclusive



Example: $f(x) = 6x^5 - 10x^3$

Domain $= (-\infty, \infty)$

Critical Points

$f'(x) = 30x^4 - 30x^2 = 30x^2(x^2-1) =$
$= 30x^2(x+1)(x-1)$

$f'(x) = 0$

$30x^2(x+1)(x-1) = 0$

$\boxed{x=0}$ c.p $\boxed{x=-1}$ c.p $\boxed{x=1}$ c.p (stationary points)

$f'(x)$ undefined
No c.p over here

$f''(x) = 120x^3 - 60x$

$f''(-1) = 120(-1)^3 - 60(-1) = -120 + 60 < 0 \Rightarrow$ f has a relative max at $x=-1$

$f''(0) = 0 \Rightarrow$ the 2nd der. test is inconclusive at $x = 0$

$f''(1) = 120 - 60 = 60 > 0 \Rightarrow$ f has a relative min at $x = 1$

$-$ $-$ f'
-1 0 1

No rel. extremum at $x=0$

$f'(1/2) = -5.625 < 0$

$f'(-1/2) = 30(-1/2)^4 - 30(-1/2)^2 = -5.625 < 0$