SIMPLE LINEAR REGRESSION (PART II)

Inferences About the slope β_1

$H_0: \beta_1 = 0$

$H_a: \beta_1 > 0$

$$\hat{\beta}_1 \pm t_{\alpha/2} * s_{\hat{\beta}_1}$$

$$t = \frac{\hat{\beta}_1}{s_{\hat{\beta}_1}}$$

$s_{\hat{\beta}_1}$ = estimated standard error of the least squares slope $\hat{\beta}_1$

$$= \frac{\hat{\beta}_1}{s/\sqrt{SS_{xx}}}$$

s = estimated standard error of the regression model

$$s = \sqrt{\frac{SSE}{n-2}}$$

SSE = sum of squares of the deviations of the y values from the regression line

$$= \sum (y - \hat{y})^2 = SS_{yy} - \hat{\beta}_1 SS_{xy}$$

x	y	x^2	y^2	xy
1	2	1	4	2
2	4	4	16	8
3	5	9	25	15
4	7	16	49	28
5	8	25	64	40
Σx	Σy	Σx^2	Σy^2	Σxy
15	26	55	158	93

$$SS_{xy} = \Sigma xy - \frac{\Sigma x \Sigma y}{n} = 15$$

$$SS_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 10$$

$$SS_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n} = 22.8$$

$$\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} = 1.5$$

$$SSE = SS_{yy} - \hat{\beta}_1 \cdot SS_{xy} = .3$$

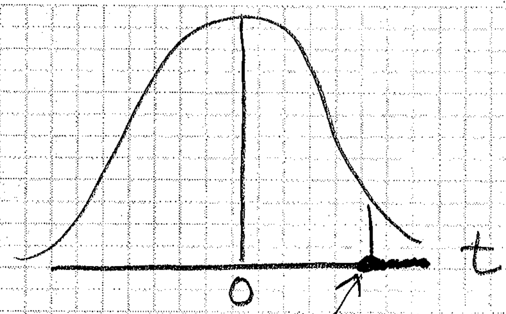
$$s = \sqrt{\frac{.3}{5-2}} \approx .316$$

$$t = \frac{1.5}{.316/\sqrt{10}} \approx 15.01$$

Rejection Region

$\alpha = .05$

$df = n-2 = 3$



RR: $t > 2.353$

$C.V = t_{.05,3} = t_\alpha$

$= 2.353$

Decision: Reject Ho

conclusion: " the data provide sufficient evidence to conclude, at $\alpha = .05$, that the slope of the regression line is greater than zero"

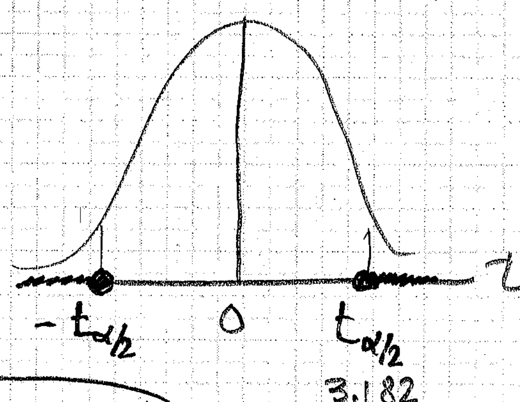
95% Confidence Interval for β_1

$\hat{\beta}_1 \pm t_{\alpha/2} * S_{\hat{\beta}_1}$

$\alpha = 1 - .95 = .05 \Rightarrow \alpha/2 = \frac{.05}{2} = .025$

$df = n-2 = 5-3=2$

CORRECTION
$5-2=3$



95% CI: $\hat{\beta}_1 \pm t_{\alpha/2} * \frac{S}{\sqrt{SS_{xx}}}$

$= 1.5 \pm 3.182 * \frac{.316}{\sqrt{10}} = (1.182 , 1.818)$

" We are 95% confident that the slope of the linear regression line is between 1.182 and 1.818