

Simple Linear Regression

y
Dependent
variable

x
Independent
variable

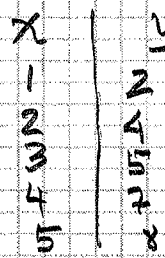
$$y = mx + b$$

↑ ↑
slope y-intercept

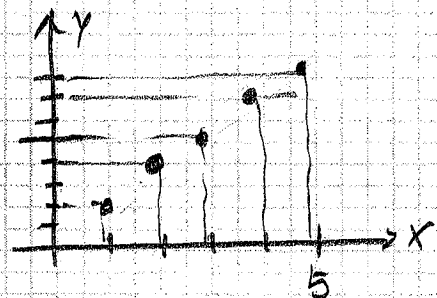
$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$$

↑ ↑
y-intercept slope

$$y = b_0 + b_1 x$$



Scatter plot



Least squares Method



| x | y | x ² | y ² | xy |
|---|---|----------------|----------------|----|
| 1 | 2 | 1 | 4 | 2 |
| 2 | 4 | 4 | 16 | 8 |
| 3 | 5 | 9 | 25 | 15 |
| 4 | 7 | 16 | 49 | 28 |
| 5 | 8 | 25 | 64 | 40 |

| Σx | Σy | Σx ² | Σy ² | Σxy |
|----|----|-----------------|-----------------|-----|
| 15 | 26 | 55 | 158 | 93 |

$a = 1.5 \rightarrow \hat{\beta}_1$

$b = 0.7 \rightarrow \hat{\beta}_0$

$r = .99 \rightarrow$ coeff of correlation

$$SS_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 93 - \frac{15 \times 26}{5} = 15$$

$$SS_{xx} = \sum x^2 - \frac{(\sum x)^2}{n} = 55 - \frac{15^2}{5} = 10$$

$$SS_{yy} = \sum y^2 - \frac{(\sum y)^2}{n} = 158 - \frac{26^2}{5} = 22.8$$

$$\bar{x} = \frac{\sum x}{n} = \frac{15}{5} = 3 \quad \bar{y} = \frac{\sum y}{n} = \frac{26}{5} = 5.2$$

$$\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{15}{10} = 1.5$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 5.2 - 1.5 \times 3 = 0.7$$

Linear Regression
Equation:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$$

$$\hat{y} = 0.7 + 1.5x$$

\nwarrow \nearrow
 $\hat{\beta}_0$ $\hat{\beta}_1$