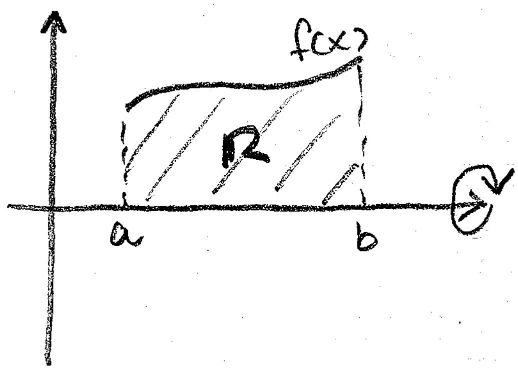
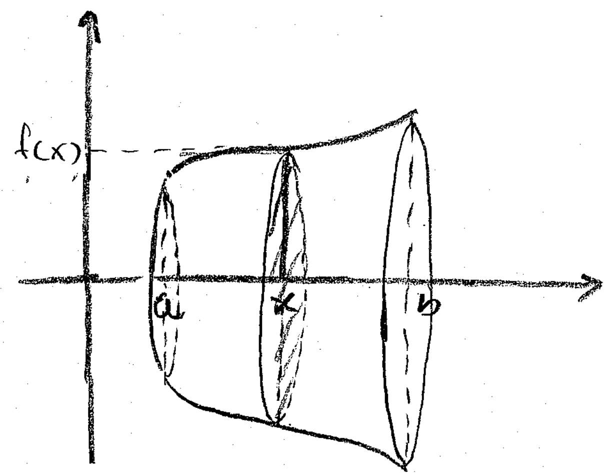
SOLIDS OF REVOLUTION

A solid of revolution is a solid that is generated by revolving a plane region around a line that is on the same plane as the region. That line is called the <u>axis of revolution</u>





Let f be continuous and non-negative on [a,b]

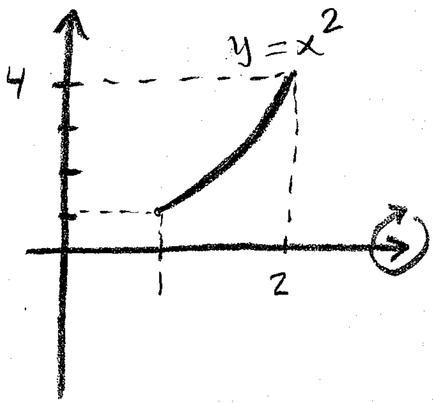
cross section is a disk

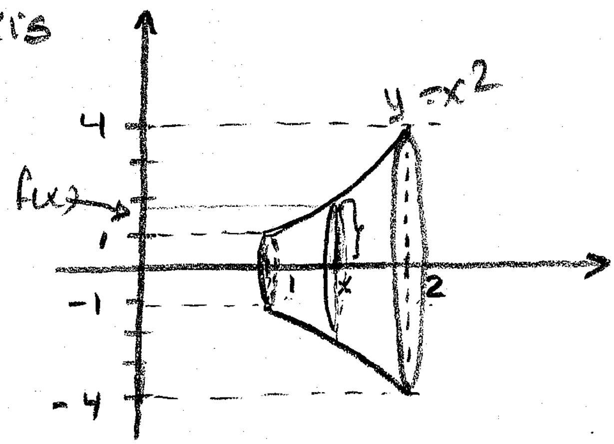
$$A(x) = \pi r^2 = \pi [f(x)]^2$$

$$V = \int_a^b A(x)\,dx = \int_a^b \pi [f(x)]^2 dx = \pi \int_a^b [f(x)]^2 dx$$

<u>Method of Disks</u>

<u>Example</u>: Find the volume of the solid generated by rotating the region under the curve $y = x^2$ over the interval $[1, 2]$, around the x-axis



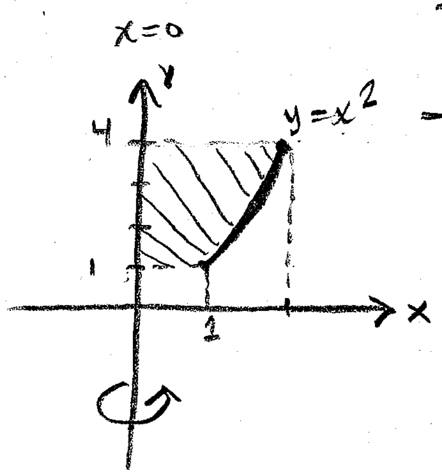


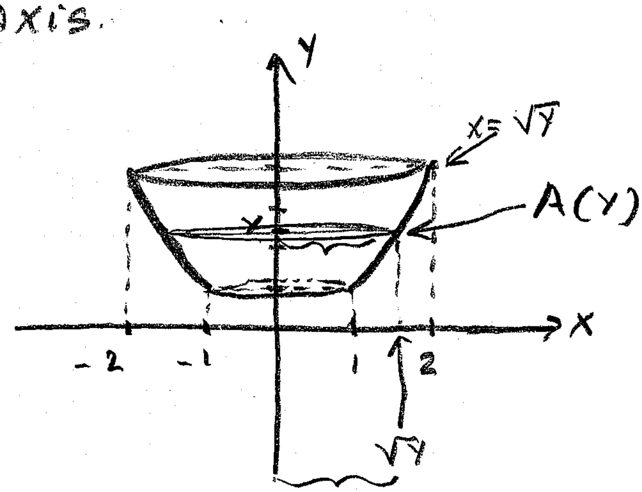
$$A(x) = \pi r^2 = \pi\,[f(x)]^2 = \pi\,(x^2)^2 = \pi x^4$$

$$V = \int_1^2 A(x)\,dx = \int_1^2 \pi x^4\,dx = \pi\,\frac{x^5}{5}\Big|_1^2$$

$$= \pi\left(\frac{2^5}{5} - \frac{1^5}{5}\right) = \frac{31\pi}{5}$$

Example: Find the volume of the solid generated when the region enclosed by $y = x^2$, $y = 1$, $y = 4$ and $x = 0$ is revolved around the y-axis.





$$V = \int_1^4 A(y)\,dy$$

$$A(y) = \pi r^2 = \pi(\sqrt{y})^2 = \pi y$$

$$V = \int_1^4 \pi y\,dy = \pi \frac{y^2}{2}\Bigg|_1^4 = \pi\left(\frac{4^2}{2} - \frac{1^2}{2}\right) = \frac{15\pi}{2}$$