Standard Deviation

Example: 2, 4, 6, 8, 10

$$\bar{x} = \frac{\Sigma x}{n} = \frac{2+4+6+8+10}{5} = 6$$

x (in)	Deviations from the mean $x - \bar{x}$	$(x-\bar{x})^2$
2	$2-6 = -4$	16
4	$4-6 = -2$	4
6	$6-6 = 0$	0
8	$8-6 = 2$	4
10	$10-6 = 4$	16
	0	40

Sample Standard Deviation $= \boxed{s = \sqrt{\frac{\Sigma (x-\bar{x})^2}{n-1}}} = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}}$

$$\sqrt{\frac{\Sigma x^2 - n\bar{x}^2}{n-1}}$$

	Mean	Std. Dev
Population	μ	σ
Sample	$\bar{x}$	s

$$s = \sqrt{\frac{40}{5-1}} = \sqrt{10} = 3.16$$

x	x^2
2	4
4	16
6	36
8	64
10	100
30	220

$$S = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}}$$

$$= \sqrt{\frac{220 - \frac{30^2}{5}}{5-1}} = 3.16$$

Given a Frequency Distribution

x	f	$f \cdot x$	$f \cdot x^2$
2	3	6	12
4	2	8	38
6	1	6	36
8	5	40	320
10	4	40	400
30	15	100	800
Σx	$\Sigma f = n$	$\Sigma(fx)$	$\Sigma(f \cdot x^2)$

$$S = \sqrt{\frac{\Sigma(f \cdot x^2) - \frac{(\Sigma fx)^2}{n}}{n-1}}$$

$$= \sqrt{\frac{800 - \frac{100^2}{15}}{14}} \approx 3.08$$

$$s = \sqrt{\frac{\Sigma x^2 - n\bar{x}^2}{n-1}} \rightarrow \sqrt{\frac{\Sigma(f\cdot x^2) - n\bar{x}^2}{n-1}}$$

$$\bar{x} = \frac{\Sigma x}{n} \quad \text{where} \quad \bar{x} = \frac{\Sigma(f\cdot x)}{n}$$

Given a Grouped Distribution

	f	Midpoints x	$f\cdot x$	$f\cdot x^2$
0 - 9	3	4.5	13.5	60.75
10 - 19	2	14.5	29	420.5
20 - 29	1	24.5	24.5	600.25
30 - 39	5	34.5	172.5	5951.25
40 - 49	4	44.5	178	7921
	$n = 15$		417.5 $\Sigma(f\cdot x)$	14,953.75 $\Sigma(f\cdot x^2)$

$$\bar{x} = \frac{\Sigma(f\cdot x)}{n} = \frac{417.5}{15} \approx 27.8$$

$$s = \sqrt{\frac{\Sigma(fx^2) - n\bar{x}^2}{n-1}} = \sqrt{\frac{14,953.75 - 15 \times 27.8^2}{14}}$$

$$\approx 15.49$$