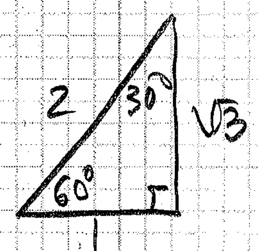
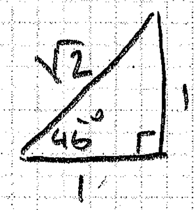
SUM AND DIFFERENCE FORMULAS

$$\boxed{\begin{aligned} \sin(\alpha+\beta) &= \sin\alpha\cos\beta + \cos\alpha\sin\beta \\ \cos(\alpha+\beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta \end{aligned}}$$

Ex: Find the sine and cosine of $75°$

$$75° = 30° + 45°$$





$$\sin 75° = \sin(\underset{\alpha}{30°} + \underset{\beta}{45°})$$

$$= \sin 30° * \cos 45° + \cos 30° * \sin 45°$$

$$= \frac{1}{2} * \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} * \frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}}$$

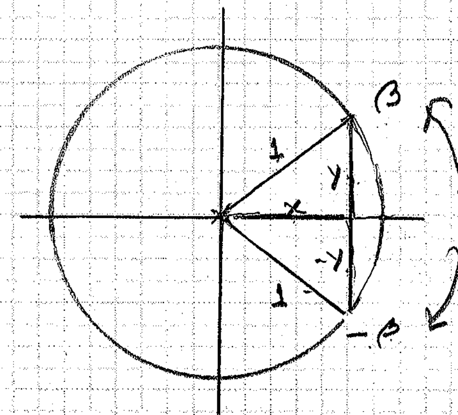
$$= \frac{(1+\sqrt{3})\sqrt{2}}{2\underbrace{\sqrt{2}\cdot\sqrt{2}}} = \frac{\sqrt{2}+\sqrt{6}}{2\cdot 2} = \frac{\sqrt{2}+\sqrt{6}}{4}$$

$$\cos 75° = \cos(\underset{\alpha}{30°} + \underset{\beta}{45°}) = \cos 30° * \cos 45° - \sin 30° * \sin 45°$$

$$= \frac{\sqrt{3}}{2} * \frac{1}{\sqrt{2}} - \frac{1}{2} * \frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{(\sqrt{3}-1)\sqrt{2}}{2\underbrace{\sqrt{2}\cdot\sqrt{2}}} = \frac{\sqrt{6}-\sqrt{2}}{2\cdot 2} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\boxed{\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta}$$



$$\sin(\alpha-\beta) = \sin(\alpha+(-\beta))$$

$$= \sin\alpha\cos(-\beta) + \cos\alpha\sin(-\beta)$$

$$\boxed{\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\cdot\sin\beta}$$

$$\cos(-\beta) = \cos\beta$$

$$\sin(-\beta) = -\sin\beta$$

$$\cos(\alpha-\beta) = \cos(\alpha+(-\beta))$$

$$= \cos\alpha\cos(-\beta) - \sin\alpha\sin(-\beta)$$

$$\boxed{\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta}$$

$$\boxed{\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\cdot\tan\beta}}$$

$$\tan(\alpha-\beta) = \tan(\alpha+(-\beta)) = \frac{\tan\alpha + \tan(-\beta)}{1 - \tan\alpha\tan(-\beta)}$$

$$\boxed{\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}}$$

Ex: Find the trigonometric functions of $\pi/12$

$$30^\circ \qquad 45^\circ \qquad 60^\circ \qquad \frac{\pi}{12}$$

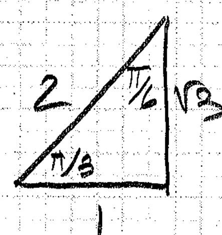
$$\frac{\pi}{6} \qquad \frac{\pi}{4} \qquad \frac{\pi}{3}$$

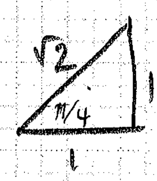
$$\frac{2\pi}{12} \qquad \frac{3\pi}{12} \qquad \frac{4\pi}{12}$$

$$\sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\ \frac{1}{\sqrt{2}}$$





$$= \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{(\sqrt{3}-1)\sqrt{2}}{2\sqrt{2}\sqrt{2}}$$

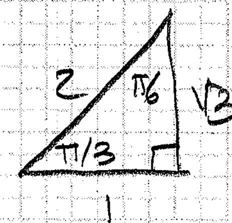
$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

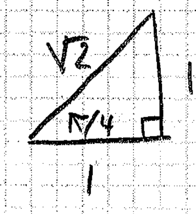
$$\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{(\sqrt{3}+1)\sqrt{2}}{2\sqrt{2}\sqrt{2}}$$

$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\tan\frac{\pi}{12} = \tan\left(\overset{\alpha}{\frac{\pi}{4}} - \overset{\beta}{\frac{\pi}{6}}\right)$$





$$= \frac{\tan\frac{\pi}{4} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{4} * \tan\frac{\pi}{6}}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$= \frac{\frac{1}{1} - \frac{1}{\sqrt{3}}}{1 + 1\cdot\frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\cancel{\sqrt{3}}} \cdot \frac{\cancel{\sqrt{3}}}{\sqrt{3}+1} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{3-\sqrt{3}-\sqrt{3}+1}{3-1} = \frac{4-2\sqrt{3}}{2}$$

$$(a+b)(a-b) = a^2 - b^2$$

$$= \frac{\cancel{2}(2-\sqrt{3})}{\cancel{2}} = 2-\sqrt{3}$$

$$\cot\frac{\pi}{12} = \frac{1}{\tan\frac{\pi}{12}} = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \ldots$$

$$\sec\frac{\pi}{12} = \frac{1}{\cos\pi/12} = \frac{2\sqrt{2}}{\sqrt{3}+1} = \ldots$$

$$\csc\frac{\pi}{12} = \frac{1}{\sin\pi/12} = \frac{2\sqrt{2}}{\sqrt{3}-1} = \ldots$$