

THE CHAIN RULE

$$y = u^4 \Rightarrow \frac{dy}{du} = 4u^3$$

$$u = 2x^3 + 5x \Rightarrow \frac{du}{dx} = 6x^2 + 5$$

$$y \rightarrow u \rightarrow x \quad \text{"chain"}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 4u^3 (6x^2 + 5) = 4(2x^3 + 5x)^3 (6x^2 + 5)$$

$$h(x) = (2x^3 + 5x)^4 \quad h'(x) \quad \begin{array}{l} f(x) = x^4 \\ g(x) = 2x^3 + 5x \end{array}$$

$$h(x) = f(g(x)) = f(2x^3 + 5x) = (2x^3 + 5x)^4$$

$$[f(g(x))] = f'(g(x)) \cdot g'(x)$$

$$h'(x) = 4(2x^3 + 5x)^3 \cdot (6x^2 + 5)$$

Ex:  $y = \left( \frac{2x+3}{4x^2-5x} \right)^5$  ← outer

$$y' = 5 \left( \frac{2x+3}{4x^2-5x} \right)^4 \cdot \left( \frac{2 \cdot (4x^2-5x) - (2x+3)(8x-5)}{(4x^2-5x)^2} \right)$$

$$\left( \frac{f}{g} \right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

Ex:  $y = u^5 \Rightarrow \frac{dy}{du} = 5u^4$

$$u = \frac{2x+3}{4x^2-5x} \Rightarrow \frac{du}{dx} = \frac{2(4x^2-5x) - (2x+3)(8x-5)}{(4x^2-5x)^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 5u^4 \cdot \left( \frac{2(4x^2-5x) - (2x+3)(8x-5)}{(4x^2-5x)^2} \right)$$

Ex:  $y = \frac{(5x-2)^3}{(3x^2+2x)^5}$

$$\left( \frac{f}{g} \right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

$$y' = \frac{3(5x-2)^2 \cdot 5(3x^2+2x)^5 - (5x-2)^3 \cdot 5(3x^2+2x)^4 \cdot (6x+2)}{[(3x^2+2x)^5]^2}$$