

THE CHI-SQUARE TEST FOR INDEPENDENCE

Goodness of Fit test

One-way tables
(only one categorical variable)

Test For Independence

Two-way Tables
(Contingency Tables)
Data is classified according
to two categorical variables

Example: Suppose we have two categorical variables:

Origin: American Latin Asian

Use of product: Uses it Does not use it

and we want to find out if the two variables are independent or dependent

Step 1 H_0 : origin and use of product are independent
 H_a : " " " are dependent

↗
(Knowing the level of one of the variables helps predict the level of the other.)

Step 2 Test statistic $\chi^2 = \sum \frac{(O-E)^2}{E}$

| | American | Latin | Asian | |
|--------------------------|-----------------------------------|-----------------------------------|-----------------------------------|-------------|
| Uses the Product | $O_{11} = 50$ $E_{11} = 42.75$ | $O_{12} = 30$ $E_{12} = 24.75$ | $O_{13} = 10$ $E_{13} = 22.50$ | $r_1 = 90$ |
| Does not use the product | $O_{21} = 45$ $E_{21} = 52.25$ | $O_{22} = 25$ $E_{22} = 30.25$ | $O_{23} = 40$ $E_{23} = 27.50$ | $r_2 = 110$ |
| | $c_1 = 95$ | $c_2 = 55$ | $c_3 = 50$ | $n = 200$ |

CORRECTED

$$E_{11} = \frac{r_1 \cdot c_1}{n} = \frac{90 \cdot 95}{200} = 42.75$$

$$E_{21} = \frac{r_2 \cdot c_1}{n} = \frac{110 \cdot 95}{200} = 52.25$$

$$E_{12} = \frac{r_1 \cdot c_2}{n} = \frac{90 \cdot 55}{200} = 24.75$$

$$E_{22} = \frac{r_2 \cdot c_2}{n} = \frac{110 \cdot 55}{200} = 30.25$$

$$E_{13} = \frac{r_1 \cdot c_3}{n} = \frac{90 \cdot 50}{200} = 22.50$$

$$E_{23} = \frac{r_2 \cdot c_3}{n} = \frac{110 \cdot 50}{200} = 27.50$$

$$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(50-42.75)^2}{42.75} + \frac{(30-24.75)^2}{24.75} + \frac{(10-22.50)^2}{22.50} + \frac{(45-52.25)^2}{52.25} + \frac{(25-30.25)^2}{30.25} + \frac{(40-27.50)^2}{27.50} = 16.89$$

Step 3 Rejection Region

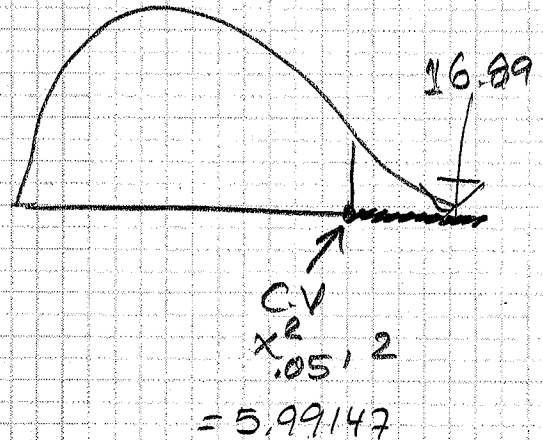
$$\alpha = .05$$

$$df = (R-1)(C-1)$$

$$R = 2 \quad C = 3$$

$$df = (2-1)(3-1) = 1 \cdot 2 = 2$$

$$RR: \chi^2 > 5.99147$$



Step 4 Decision: Reject H_0

Step 5 conclusion: "The data provide sufficient evidence, at $\alpha = .05$, to conclude that Origin and Use of Product are dependent"