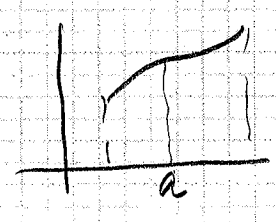
THE DEFINITE INTEGRAL

$$A = \lim_{n \to +\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

$$A = \lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} f(x_k^*)\Delta x_k = \int_a^b f(x)\,dx$$



Properties

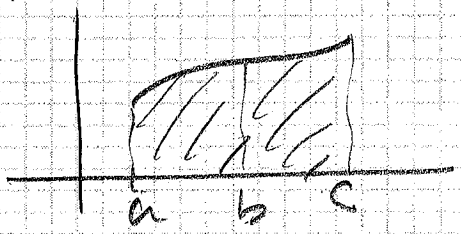
1) $\int_a^a f(x)\,dx = 0$

2) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

3) $\int_a^b k f(x)\,dx = k\int_a^b f(x)\,dx$

4) $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

5) $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$



Fundamental theorem of Calculus (Part I)

Informally, if $\int f(x)\,dx = F(x) + C$

then $\int_a^b f(x)\,dx = F(b) - F(a) = F(x)\Big]_a^b$

Example: Calculate $\int_0^1 x^2\,dx =$

$= \frac{x^3}{3}\Big]_0^1 = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3}$

Example Calculate $\int_{-2}^{2} (4x^2 - 3x + 6)\,dx =$

$= 4\frac{x^3}{3} - 3\frac{x^2}{2} + 6x\Big]_{-2}^{2} =$

$= \underbrace{4 * \frac{2^3}{3} - 3 * \frac{2^2}{2} + 6 * 2} - \left(4 * \frac{(-2)^3}{3} - 3\frac{(-2)^2}{2} + 6(-2)\right)$

$= \frac{136}{3}$

Example: Calculate $\int_0^{\pi/3} \sec^2\theta\,d\theta = \tan\theta\Big]_0^{\pi/3}$

$= \tan(\pi/3) - \tan(0)$

$= \sqrt{3} - 0$

$= \sqrt{3}$