THE FUNDAMENTAL THEOREM OF CALCULUS

$$\int_a^b f(x)\,dx = F(b) - F(a) = F(x)\Big]_a^b$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Ex: $y = \int_a^x \cos t\,dt$ find $\frac{dy}{dx}$

$$\frac{d}{dx}\int_a^x \cos t\,dt = \cos x$$

Ex: $y = \int_x^3 4t\sin t\,dt$ find $\frac{dy}{dx}$

$$\frac{d}{dx}\int_x^3 4t\sin t\,dt = \frac{d}{dx}\left(-\int_3^x 4t\sin t\,dt\right)$$

$$= -4x\sin x$$

Ex: $y = \int_0^{x^3} \cos t\,dt$ $\frac{dy}{dx}$

$$y = \int_0^u \cos t\,dt \quad \text{and} \quad u = x^3$$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \frac{d}{du}\int_0^u \cos t\,dt \cdot \frac{d}{dx}(x^3) =$$

$$= \cos u \cdot 3x^2 = \cos(x^3)\cdot 3x^2 = 3x^2\cos x^3$$

$$y = \int_0^{x^3} \cos t \, dt \qquad \frac{dy}{dx}$$

$$y = \int_1^{u} \cos t \, dt \quad \text{and} \quad u = x^3$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{d}{du} \int_1^{u} \cos t \, dt \cdot \frac{du}{dx}$$

$$= \cos u \cdot 3x = \cos(x^3)\, 3x = 3x \cos(x^3)$$