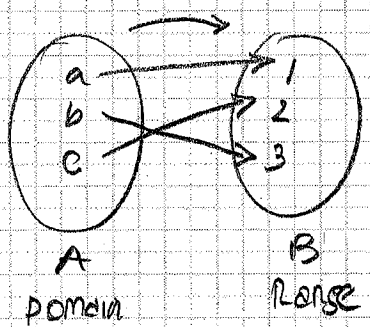
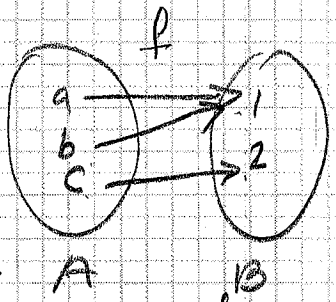
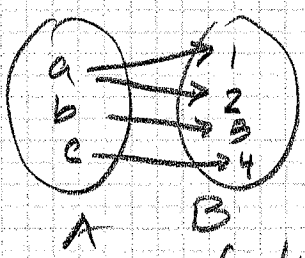
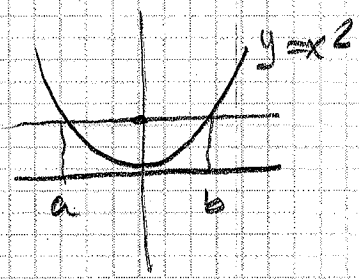
THE INVERSE SINE FUNCTION

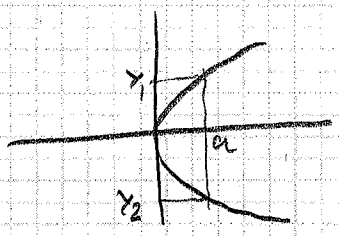
Venn Diagrams

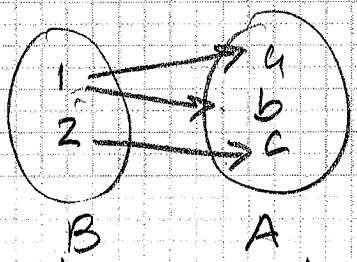


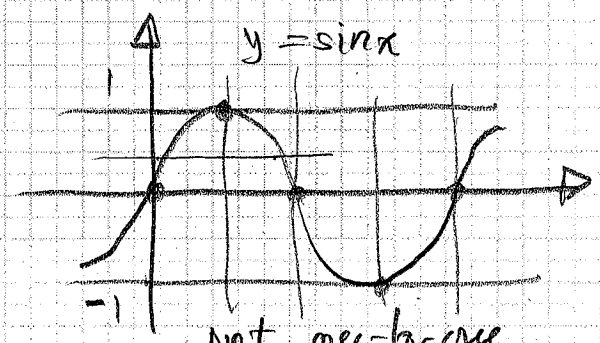


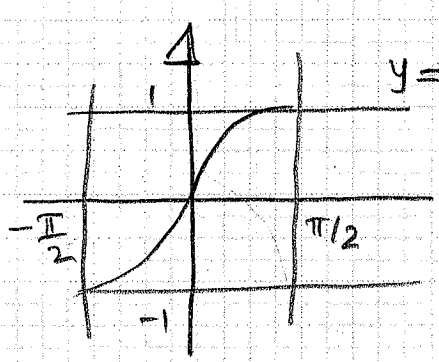




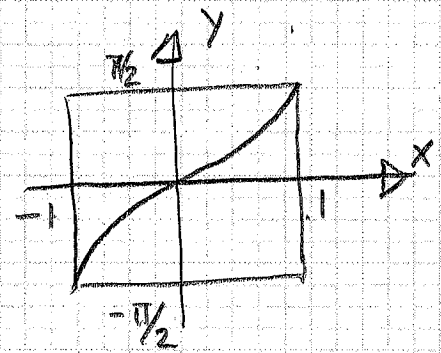






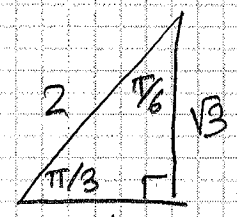


y = sin x restricted to $[-\frac{\pi}{2}, \pi/2]$

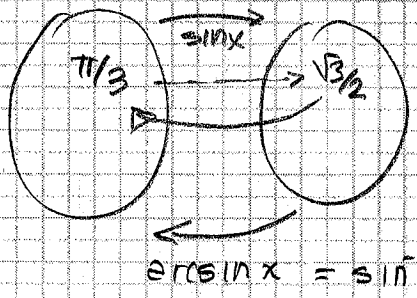


$y = \sin^{-1} x$

$y = \arcsin x$

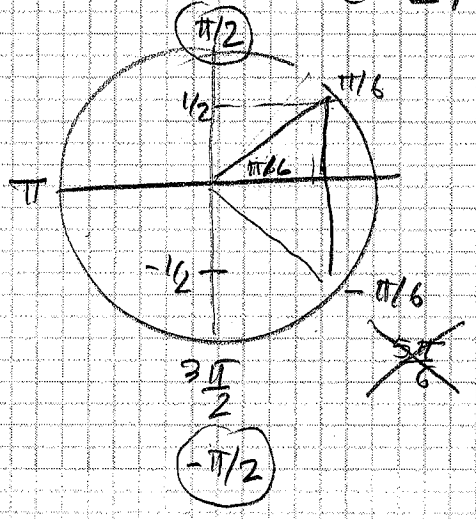


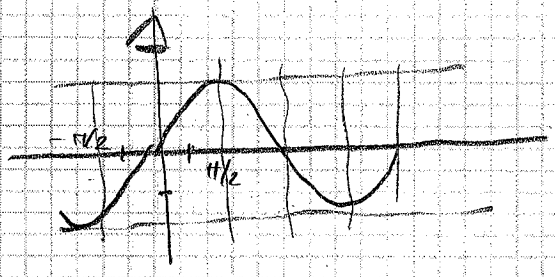
$\sin(\pi/3) = \frac{\sqrt{3}}{2} \Rightarrow \arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$



$\arcsin x = \sin^{-1} x$

Ex: Find $\sin^{-1}\left(-\frac{1}{2}\right) = -\pi/6$





$f^{-1}(f(x)) = x$

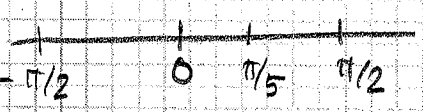
$f(f^{-1}(x)) = x$

$\sin x \qquad \sin^{-1} x$

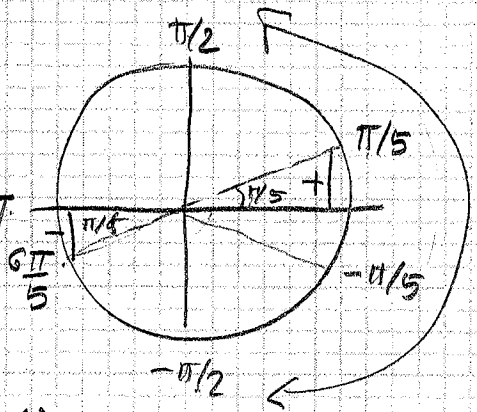
$\sin^{-1}(\sin(x)) = x \quad \text{if} \quad -\frac{\pi}{2} \le x \le \pi/2$

$\sin(\sin^{-1}(x)) = x \quad \text{if} \quad -1 \le x \le 1$

Ex: $\sin^{-1}\left(\sin\left(\frac{\pi}{5}\right)\right) = \frac{\pi}{5}$ $\quad -\frac{\pi}{2} \leq \frac{\pi}{5} \leq \frac{\pi}{2}$



Ex: $\sin^{-1}\left(\sin\left(\frac{6\pi}{5}\right)\right) \neq \frac{6\pi}{5}$? NO



$\sin\left(\frac{6\pi}{5}\right) = -\sin\left(\frac{\pi}{5}\right) = \sin\left(-\frac{\pi}{5}\right)$

sine is an odd function: $-\sin(x) = \sin(-x)$

$\sin^{-1}\left(\sin\left(\frac{6\pi}{5}\right)\right) = \sin^{-1}\left(\sin\left(-\frac{\pi}{5}\right)\right) = -\pi/5$