THE NUMBER e

e is defined as the limit

$$\boxed{e = \lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^x}$$

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Natural logs.

x	$(1+\frac{1}{x})^x$
1	$(1+\frac{1}{1})^1 = 2$
10	$(1+\frac{1}{10})^{10} = 2.59374246\ldots$
1000	$(1+\frac{1}{1000})^{1000} = 2.7169239\ldots$
100,000	$(1+\frac{1}{100{,}000})^{100{,}000} = 2.71826\,8237\ldots$
10,000,000	$(1+\frac{1}{10{,}000{,}000})^{10{,}000{,}000} = 2.7182816\ldots$

$$\frac{1}{x} = u \Rightarrow x = \frac{1}{u}$$

$$e = \lim_{u\to 0} (1+u)^{\frac{1}{u}}$$

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$$1 + \frac{1}{x} = \frac{1}{u} \Rightarrow \frac{1}{x} = \frac{1}{u} - 1 = \frac{1-u}{u} \Rightarrow x = \frac{u}{1-u}$$

$$e = \lim_{u\to 1} \left(\frac{1}{u}\right)^{\frac{u}{1-u}}$$

$$\boxed{e = \lim_{x\to 1} \left(\frac{1}{x}\right)^{\frac{x}{1-x}}}$$