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THE SIGN TEST

The Sign Test is a non-parametric test about the median, η, of a population

	mean	std Dev	Proportion	
population	μ	σ	p	← parameters
sample	$\bar{x}$	s	$\hat{p}$	← statistics

t test

- about the mean (μ)
- population must be normal
- parametric test
  uses estimates of population parameter

Sign test "eta"

- about the median (η)
- population must be continuous
- non-parametric test
  doesn't use any estimates of population parameters

Example: Given the following data

7.4, 15.6, 14.3, 11.2, 9.4, 3.9, 11.6, 8.4, 12.5, 6.8
 −, +, +, +, +, −, +, +, +, −

do the following sign test using $\alpha = .05$

Step 1

$H_0: \eta = 8$

$H_a: \eta > 8$

$n = 10$

(If any observation is equal to 8, eliminate it and decrease n by one)

Step 2 Test statistic

S_+ = # of sample observations greater than 8 = 7

we will reject H_0 if S_+ is "sufficiently" large

Step 3 Pvalue

"The number of observations greater than 8" is a Binomial random variable, X, and S_+ is the observed value of X

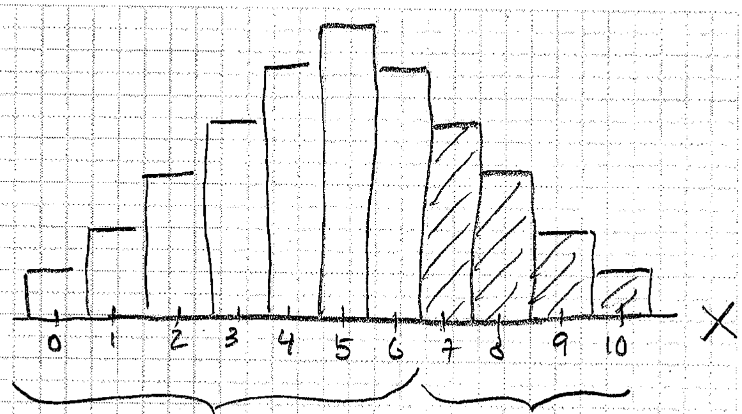
success = greater than 8

$n = 10$ trials $p = .5$ (always)

Pvalue = $P(X \geq 7)$

$= 1 - P(X \leq 6)$

table
$n = 10$
$p = .5$ (always)
$x = 6$



$P(X \leq 6) + P(X \geq 7) = 1$

$= 1 - .828 = .172$

Step 4 Decision

Since pval $= .172 > \alpha = .05$, we fail to reject Ho

Step 5 Conclusion

The data provide <u>insufficient</u> evidence, at $\alpha = .05$, to conclude that the median is greater than 8

Example 2

7.4 , 5.6 , 4.3 , 11.2 , 7.5 , 3.9 , 4.6 , 6.4 , 2.5 , 6.8
– – – + – – – – – –

$n = 10$

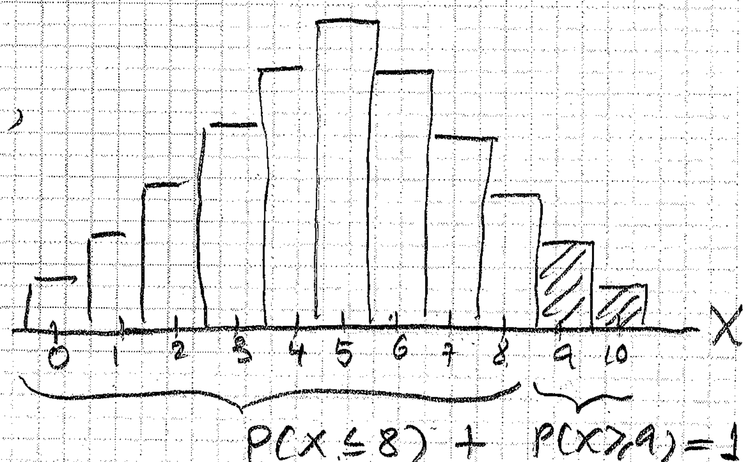
Step 1
$H_0: \eta = 8$
$H_a: \eta < 8$
$\alpha = .05$

Step 2 Test statistic

$S_- =$ # of sample observations less than 8 $= 9$

Step 3 Pval

The Binomial random variable, X is now, "the number of observations less than 8", and S_- is the observed value of X



$$\text{Pvalue} = P(X \geq 9)$$

$$= 1 - P(X \leq 8)$$

$n = 10$
$p = .5$ (always)
$x = 8$

$$= 1 - .989 = .011$$

Step 4 Decision: Reject H_0

Step 5 Conclusion: "the data provide sufficient evidence to conclude, at $\alpha = .05$, that $\eta < 8$"

Example 3.

7.4, 5.6, 4.3, 11.2, 7.5, 3.9, 4.6, 6.4, 2.5, 6.8

Step 1
$H_0: \eta = 8$ $\alpha = .05$
$H_a: \eta \neq 8$

Step 2 test stat

$S_{max} = \max(S_+, S_-) = S_- = 9$

Step 3 $P\text{value} = 2P(X \geq S_{max}) = 2P(X \geq 9)$

$P(X \geq 9) = 1 - P(X \leq 8) = 1 - .989 = .011$

$P\text{val} = 2 \times .011 = .022 < \alpha = .05$

Step 4 Decision: Reject H_0

Example 4 (Normal Approximation to the sign test)
for $n \geq 10$

7.4, 5.6, 4.3, 11.2, 7.6, 3.9, 4.6, 6.4, 2.5, 6.8 $\quad n = 10$

Step 1

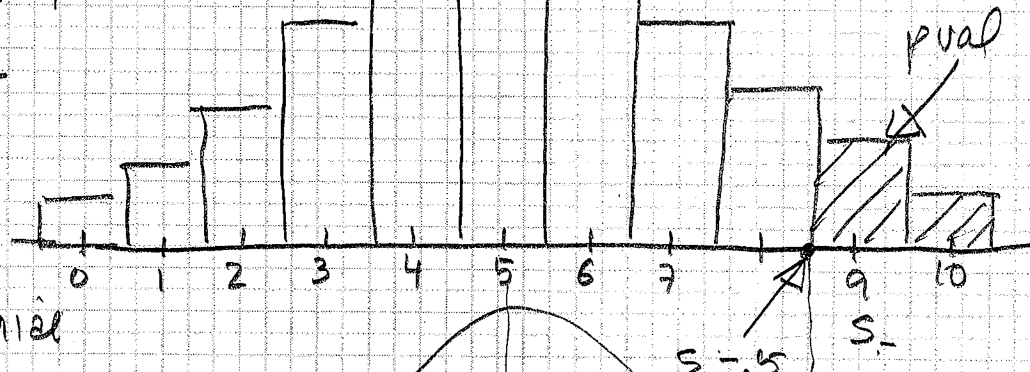
$H_0: \eta = 8$

$\alpha = .05$

$H_a: \eta < 8$

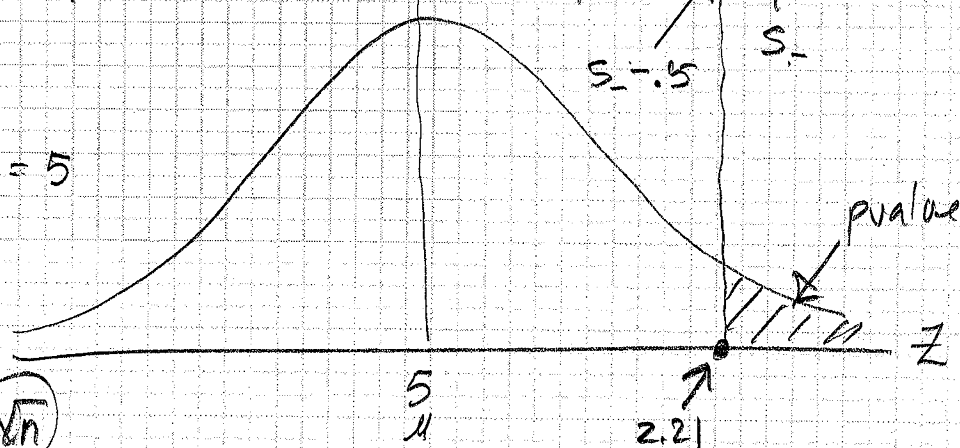
$S_- = 9$

Step 2 Test stat



For the Binomial Distribution

$\mu = n \cdot p = .5n = 5$



$\sigma = \sqrt{npq} =$

$= \sqrt{.5 * .5 * n}$

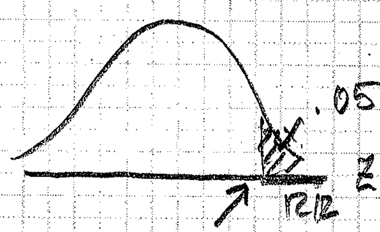
$= \sqrt{.25n} = .5\sqrt{n}$

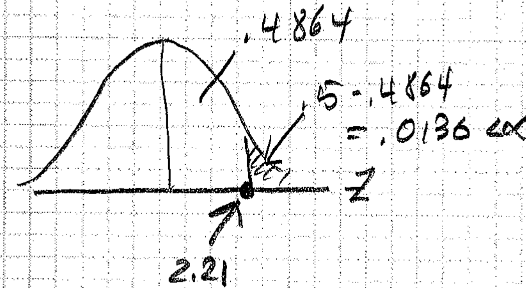
$= .5\sqrt{10}$

$$Z = \frac{x-\mu}{\sigma} = \boxed{\frac{(S_- - .5) - .5n}{.5\sqrt{n}}} = \frac{(9-.5) - .5 * 10}{.5\sqrt{10}} = 2.21$$

Step 3 Rejection Region or P value

$\alpha = .05$





RR: $Z > 1.645$

$H_0: \eta = \eta_0$	$H_0: \eta = \eta_0$	$H_0: \eta = \eta_0$
$H_a: \eta > \eta_0$	$H_a: \eta < \eta_0$	$H_a: \eta \neq \eta_0$
RR: $Z > Z_\alpha$	RR: $Z > Z_\alpha$	RR: $Z > Z_{\alpha/2}$
S_+ "large"	S_- "large"	S_{max}