

THE WILCOXON RANK SUM TEST (PART II)

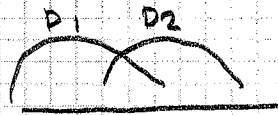
When both sample sizes n_1 and n_2 are greater than or equal than 10, the sampling distribution of T_1 can be approximated by a normal distribution with

$$\mu_{T_1} = \frac{n_1(n_1+n_2+1)}{2} \quad \text{and} \quad \sigma_{T_1} = \sqrt{\frac{n_1 n_2 (n_1+n_2+1)}{12}}$$

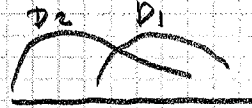
H_0 : D_1 and D_2 are identical

Step 1

H_a : 1) D_1 is shifted left of D_2



2) D_1 is shifted right of D_2



3) D_1 is shifted left or right of D_2

Step 2 Test Statistic

$$Z = \frac{\bar{X} - \mu_0}{\sigma_{\bar{X}}} = \frac{T_1 - \frac{n_1(n_1+n_2+1)}{2}}{\sqrt{\frac{n_1 n_2 (n_1+n_2+1)}{12}}}$$

Step 3 Rejection Region

1) $Z < Z_{\alpha}$

2) $Z > Z_{\alpha}$

3) $Z > Z_{\alpha/2}$
or

$Z < -Z_{\alpha/2}$

Example Let's suppose we have the following two random samples with sizes $n_1 = 10$ and $n_2 = 11$

<u>Sample 1</u>	<u>Ranks</u>	<u>Sample 2</u>	<u>Ranks</u>
12.5	8	15.1	19
13.0	12	13.4	14
10.2	2.5	14.6	17
9.0	1	13.0	12
11.4	5	15.2	20
12.6	9	15.4	21
11.7	6	14.5	16
12.2	7	13.0	12
10.5	4	14.8	18
10.2	2.5	12.8	10
	<u> </u>	13.7	<u>15</u>
	$T_1 = 57$		

9	10.2	10.2	10.5	11.4	11.7	12.2	12.5	12.6	12.8	13	13	13
1	2	3	4	5	6	7	8	9	10	11	12	13
1	2.5	2.5	4	5	6	7	8	9	10	12	12	12
/ / / / / / / / / / / / / /												
13.4	13.7	14.5	14.6	14.8	15.1	15.2	15.4					
14	15	16	17	18	19	20	21					
14	15	16	17	18	19	20	21					

(Step 1) H_0 : D_1 and D_2 are identical

H_a : D_1 is shifted left of D_2

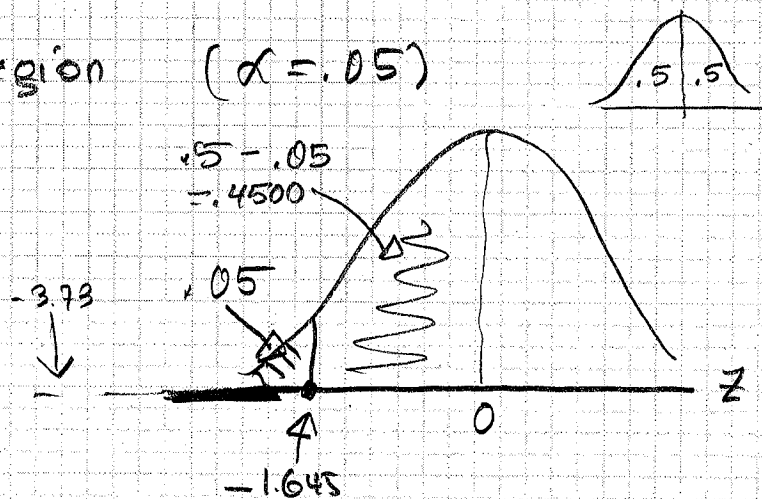
(Step 2) Test statistic

$$Z = \frac{T_1 - \frac{n_1(n_1+n_2+1)}{2}}{\sqrt{\frac{n_1 n_2 (n_1+n_2+1)}{12}}} = \frac{57 - \frac{10(10+11+1)}{2}}{\sqrt{\frac{10 \cdot 11(10+11+1)}{12}}}$$

$$= -3.73$$

(Step 3) Rejection Region ($\alpha = .05$)

look-up .4500
in the body of
the table



$$RR: Z < -1.645$$

(Step 4) Decision: Reject H_0

(Step 5) Conclusion: "The data provide sufficient evidence to conclude that distribution D_1 is shifted to the left of distribution D_2 (at $\alpha = .05$)"