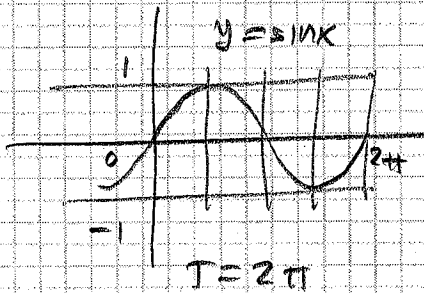


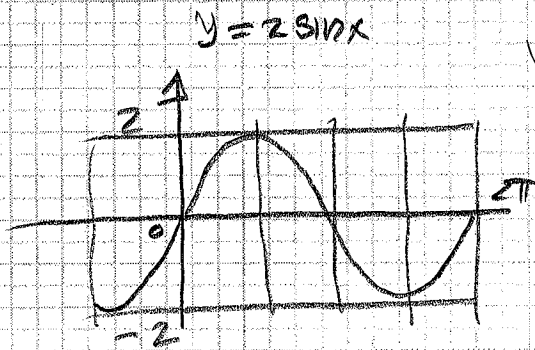
TRANSFORMATIONS: stretches, compressions, reflections
phase shifts

Ex: Graph $y = 2 \sin x$



Domain = \mathbb{R}
= $(-\infty, \infty)$

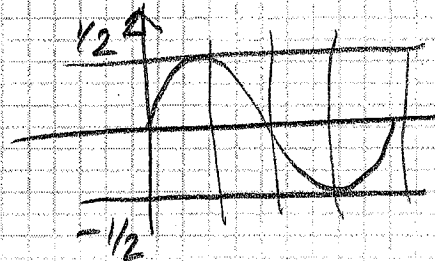
$A = 1$



Domain = \mathbb{R}

$A = 2$

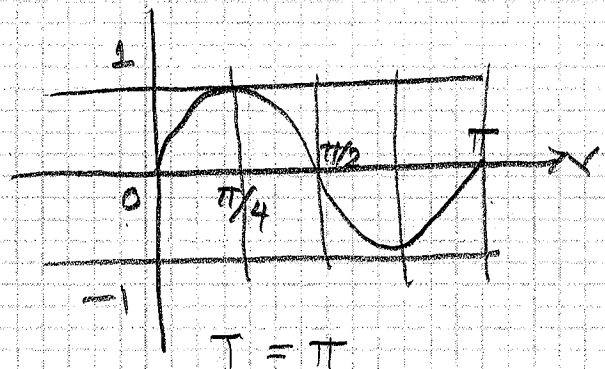
Ex: Graph $y = \frac{1}{2} \sin x$



Ex: Graph = $y = \sin(2x)$

x	$2x$	$\sin(2x)$
0	0	0
$\pi/4$	$\pi/2$	1
$\pi/2$	π	0
$3\pi/4$	$3\pi/2$	-1
π	2π	0

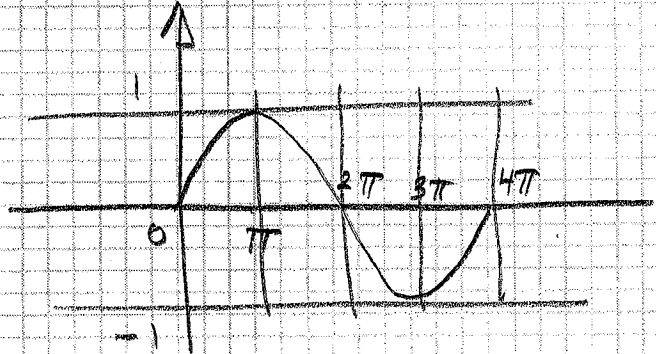
$2x = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{4}$



horizontal compression

Ex: $y = \sin\left(\frac{1}{2} \cdot x\right)$

x	$x/2$	$\sin(x/2)$
0	0	0
π	$\pi/2$	1
2π	π	0
3π	$3\pi/2$	-1
4π	2π	0



horizontal stretch

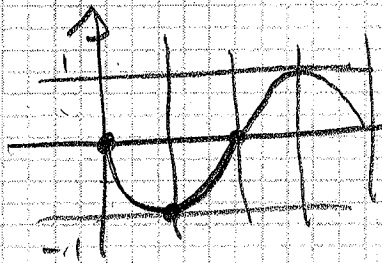
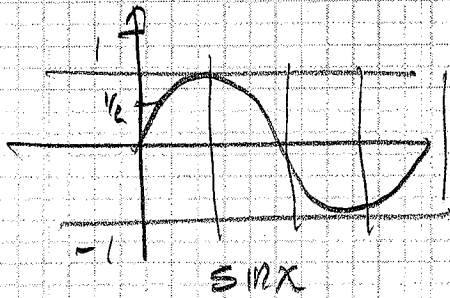
$$\frac{x}{2} = \frac{\pi}{2} \Rightarrow x = \frac{2\pi}{2} = \pi$$

$$T = 4\pi$$

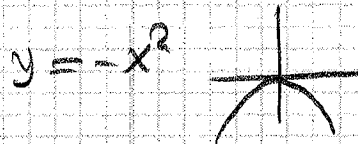
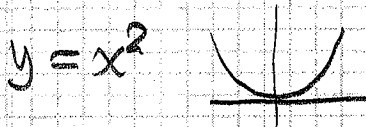
$$\frac{x}{2} = \pi \Rightarrow x = 2\pi$$

$$A = 1$$

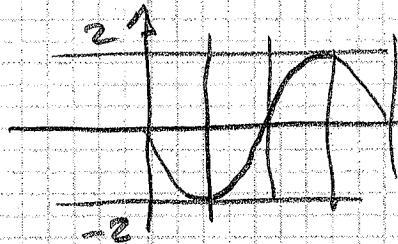
Ex: $y = -\sin x$



reflection on the x-axis

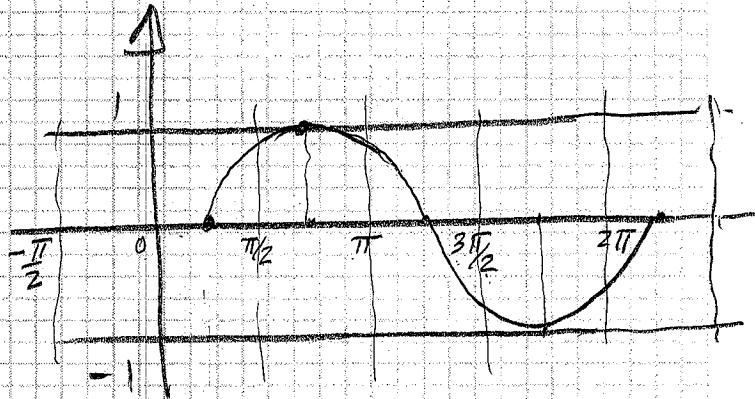


$y = -2 \sin x$



Ex: Graph $y = \sin(x - \pi/4)$

x	$x - \pi/4$	$\sin(x - \pi/4)$
$\pi/4$	0	0
$3\pi/4$	$\pi/2$	1
$5\pi/4$	π	0
$7\pi/4$	$3\pi/2$	-1
$9\pi/4$	2π	0



Phase shift = $\pi/4$

$$x - \frac{\pi}{4} = 0 \Rightarrow x = \frac{\pi}{4}$$

$$x - \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$$

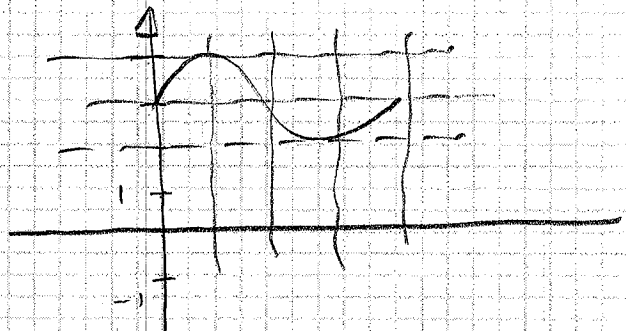
$$x - \frac{\pi}{4} = \pi \Rightarrow x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

$$x - \frac{\pi}{4} = \frac{3\pi}{2} \Rightarrow x = \frac{3\pi}{2} + \frac{\pi}{4} = \frac{7\pi}{4}$$

$$x - \frac{\pi}{4} = 2\pi \Rightarrow x = 2\pi + \frac{\pi}{4} = \frac{9\pi}{4}$$

Ex: $y = \sin(x) + 3$

vertical
shift (up)



$$y = \sin(Bx) \quad T = \frac{2\pi}{B}$$

$$y = \sin(x - \phi) \quad \text{Phase shift} = \phi$$

$$y = \sin(2x - \pi/4)$$

X	$2x - \pi/4$	$\sin(2x - \pi/4)$
	0	0
	$\pi/2$	1
	π	0
	$3\pi/2$	-1
	2π	0

$$2x - \frac{\pi}{4} = 0 \Rightarrow 2x = \frac{\pi}{4} \Rightarrow x = \frac{\pi/4}{2} = \text{Phase shift}$$

$$y = \sin(Bx - \phi) \quad \text{Phase shift} = \frac{\phi}{B}$$

$$y = A \sin(Bx - \phi) + C \quad \leftarrow \text{Vertical shift}$$

Amplitude = $|A|$

Phase shift = $\frac{\phi}{B}$

$T = \frac{2\pi}{B}$