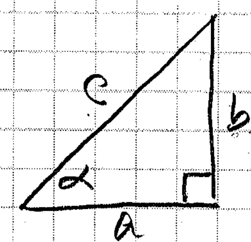
TRIGONOMETRIC IDENTITIES

Equation $x^2+5x+6=0$ $x=-3,-2$

Identity $(a+b)^2=a^2+2ab+b^2$



Reciprocal Identities

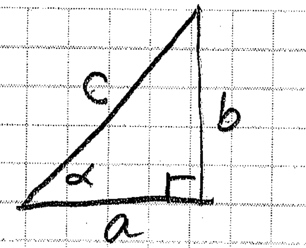
$\sin\alpha = \dfrac{b}{c}$

$\csc\alpha = \dfrac{c}{b}$

$\Rightarrow \boxed{\sin\alpha = \dfrac{1}{\csc\alpha}}$

$\boxed{\cos\alpha = \dfrac{1}{\sec\alpha}}$ $\boxed{\tan\alpha = \dfrac{1}{\cot\alpha}}$

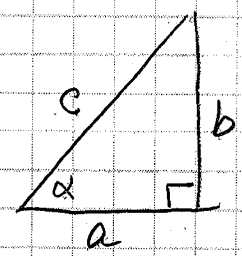
$$\boxed{\tan\alpha = \frac{\sin\alpha}{\cos\alpha}}$$



$$\frac{\sin\alpha}{\cos\alpha} = \frac{b/c}{a/c} = \frac{b}{\cancel{c}} \cdot \frac{\cancel{c}}{a} = \frac{b}{a} = \tan\alpha$$

$$\boxed{\cot\alpha = \frac{\cos\alpha}{\sin\alpha}}$$

$$\boxed{\sin^2\alpha + \cos^2\alpha = 1}$$

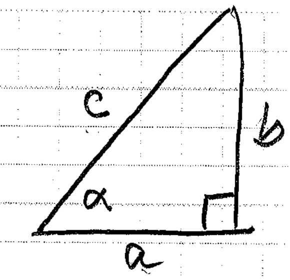


$$\sin^2\alpha = (\sin\alpha)^2$$

$$\left(\frac{b}{c}\right)^2 + \left(\frac{a}{c}\right)^2 = \frac{b^2}{c^2} + \frac{a^2}{c^2} = \frac{a^2+b^2}{c^2} = \frac{c^2}{c^2} = 1$$

$$\boxed{\tan^2\alpha + 1 = \sec^2\alpha}$$

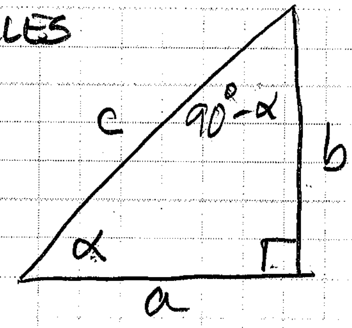
$$\boxed{\cot^2\alpha + 1 = \csc^2\alpha}$$



$$\left(\frac{b}{a}\right)^2 + 1 = \frac{b^2}{a^2} + \frac{1}{1} = \frac{b^2 + a^2}{a^2} = \frac{c^2}{a^2} =$$

$$= \left(\frac{c}{a}\right)^2 = \sec^2\alpha$$

COMPLEMENTARY ANGLES



$$\boxed{\begin{aligned} \sin\alpha &= \cos(90^\circ-\alpha) \\ \tan\alpha &= \cot(90^\circ-\alpha) \\ \sec\alpha &= \csc(90^\circ-\alpha) \end{aligned}}$$

$$\frac{b}{c} = \frac{b}{c}$$

EXAMPLE: Let θ be an acute angle and $\sin\theta = \frac{1}{2}$. Find the remaining trigonometric functions of θ

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{1/2} = 2$$

$$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1 \Rightarrow$$

$$\Rightarrow \frac{1}{4} + \cos^2\theta = 1 \Rightarrow \cos^2\theta = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\Rightarrow \cos\theta = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\sqrt{3}/2} = 1 \cdot \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\tan^2\theta + 1 = \sec^2\theta \Rightarrow \tan^2\theta + 1 = \frac{4}{3}$$

$$\Rightarrow \tan^2\theta = \frac{4}{3} - 1 = \frac{1}{3} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{1/\sqrt{3}} = \sqrt{3}$$

PROVING IDENTITIES

$$\frac{1+\tan\theta}{1-\tan\theta} = \frac{\cot\theta+1}{\cot\theta-1}$$

Proof

$$\frac{1+\tan\theta}{1-\tan\theta} = \frac{\frac{1}{1}+\frac{\sin\theta}{\cos\theta}}{\frac{1}{1}-\frac{\sin\theta}{\cos\theta}} = \frac{\frac{\cos\theta+\sin\theta}{\cos\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$\frac{\cos\theta+\sin\theta}{\cancel{\cos\theta}}\cdot\frac{\cancel{\cos\theta}}{\cos\theta-\sin\theta} = \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$$

$$\frac{\frac{\cos\theta+\sin\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\sin\theta}} = \frac{\frac{\cos\theta}{\sin\theta}+\frac{1}{1}}{\frac{\cos\theta}{\sin\theta}-\frac{1}{1}} = \frac{\cot\theta+1}{\cot\theta-1}$$