TRIGONOMETRIC INTEGRALS. PART II

Reduction Formulas

$$\int \sin^n x\, dx = -\frac{1}{n}\sin^{n-1}x \cdot \cos x + \frac{n-1}{n}\int \sin^{n-2}x\, dx$$

$$\int \cos^n x\, dx = \frac{1}{n}\cos^{n-1}x\, \sin x + \frac{n-1}{n}\int \cos^{n-2}x\, dx$$

EX: $\int \sin^4 x\, dx = -\frac{1}{4}\sin^3 x \cdot \cos x + \frac{3}{4}\int \sin^2 x\, dx$

$= -\frac{1}{4}\sin^3 x \cos x + \frac{3}{4}\left[-\frac{1}{2}\sin x \cos x + \frac{1}{2}\int \sin^0 x\, dx\right.$

NOTE $\sin^0 x = (\sin x)^0 = 1$

$= -\frac{1}{4}\sin^3 x \cos x + \frac{3}{4}\left[-\frac{1}{2}\sin x \cos x + \frac{1}{2}\int dx\right]$

$= -\frac{1}{4}\sin^3 x \cos x + \frac{3}{4}\left[-\frac{1}{2}\sin x \cos x + \frac{1}{2}x\right] + C$

$= -\frac{1}{4}\sin^3 x \cos x - \frac{3}{8}\sin x \cos x + \frac{3}{8}x + C$

$\frac{3}{8}x - \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + C =$

$= \frac{3}{8}x - \frac{1}{4}\cdot 2\sin x \cos x + \frac{1}{32}\cdot 2\sin(2x)\cos(2x) + C$

$= \frac{3}{8}x - \frac{1}{4}\, 2\sin x \cos x + \frac{1}{32}\cdot 2\cdot 2\sin x \cos x\,(1 - 2\sin^2 x) + C$

Next: $\int \sin^m x \cos^n x\, dx$ $\quad \int \tan^n x\, dx$ $\quad \int \sec^n x\, dx$

$\int \tan^m x \sec^n x\, dx$ more reduction formulas ...