TRIGONOMETRIC INTEGRALS. PART IV

Integrals of Powers of Tangent and Secant

$$\int \tan^n x\,dx \qquad \int \sec^n x\,dx$$

Ex 1: $\boxed{\int \tan x\,dx} = \int \frac{\sin x}{\cos x}\,dx = \int \frac{1}{\cos x}\cdot \sin x\,dx$

$u = \cos x$
$du = -\sin x\,dx$

$$= -\int \frac{1}{u}\,du = -\ln|u| + C = -\ln|\cos x| + C$$

$\cos x = \frac{1}{\sec x} = \sec^{-1} x$

$$= -\ln|\sec^{-1} x| + C$$

$\log_b x^n = n\log_b x$

$$= \boxed{\ln|\sec x| + C}$$

Ex 2: $\boxed{\int \sec x\,dx} = \int \sec x \left(\frac{\sec x + \tan x}{\sec x + \tan x}\right) dx$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\,dx = \int \frac{1}{u}\,du = \ln|u| + C$$

$u = \sec x + \tan x$
$du = (\sec^2 x + \sec x \tan x)\,dx$

$$= \boxed{\ln|\sec x + \tan x| + C}$$

Ex 3: $\int \tan^2 x\,dx = \int (\sec^2 x - 1)\,dx =$

$1 + \tan^2 x = \sec^2 x$

$\int \sec^2 x\,dx - \int dx$

$= \tan x - x + C$

Ex 4: $\int \sec^2 x\,dx = \tan x + C$

Ex 5: $\int \tan^3 x\,dx = \int \tan^2 x \cdot \tan x\,dx$

$= \int (\sec^2 x - 1)\tan x\,dx = \int \sec^2 x \tan x\,dx - \int \tan x\,dx$

$= \int \tan x \cdot \sec^2 x\,dx - \int \tan x\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

$= \int u\,du - \int \tan x\,dx$

$= \frac{u^2}{2} - \int \tan x\,dx = \frac{1}{2}\tan^2 x - \ln|\sec x| + C$

Ex 6: $\int \sec^3 x\,dx = \int \sec x \cdot \sec^2 x\,dx = uv - \int v\,du$

$u = \sec x$ $\quad dv = \sec^2 x\,dx$

$du = \sec x \tan x\,dx$ $\quad v = \tan x$

$$I = \sec x \tan x - \int \sec x \tan^2 x\,dx$$

$$= \sec x \tan x - \int \sec x\,(\sec^2 x - 1)\,dx$$

$$= \sec x \tan x - \int \sec^3 x\,dx + \int \sec x\,dx$$

$$2I = \sec x \tan x + \ln|\sec x + \tan x| + C$$

$$I = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C'$$

NEXT VIDEO: Reduction formulas

$$\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$$

$$\int \sec^n x\,dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$$