

TRIGONOMETRIC INTEGRALS. PART VIIntegrals of Products of Tangents and Secants

$$\int \tan^m x \sec^n x dx$$

$$\text{Ex 1: } \int \tan^5 x \sec^3 x dx \quad m \text{ is odd}$$

$$\text{Ex 2: } \int \tan^4 x \sec^4 x dx \quad n \text{ is even}$$

$$\text{Ex 3: } \int \tan^4 x \sec^3 x dx$$

$$\text{Ex 1: } \int \tan^5 x \sec^3 x dx$$

$$= \int \tan^4 x \sec^2 x \sec x \tan x dx$$

$$= \int (\sec^2 x - 1)^2 \sec^2 x \sec x \tan x dx$$

$$\left. \begin{array}{l} u = \sec x \\ du = \sec x \tan x dx \end{array} \right\} = \int (u^2 - 1)^2 u^2 du = \dots$$

$$\text{Ex 2: } \int \tan^4 x \sec^4 x dx = \int \tan^4 x \sec^2 x \sec^2 x dx$$

$$= \int \tan^4 x (\tan^2 x + 1) \sec^2 x dx$$

$$\left. \begin{array}{l} u = \tan x \\ du = \sec^2 x dx \end{array} \right\} = \int u^4 (u^2 + 1) du = \dots$$

$$\text{EX 3: } \int \tan^4 x \sec^3 x \, dx$$

$m$  is not odd  
 $n$  is not even

$$\tan^2 x = \sec^2 x - 1$$

$$= \int (\sec^2 x - 1)^2 \sec^3 x \, dx$$

$$= \int (\sec^4 x - 2\sec^2 x + 1) \sec^3 x \, dx$$

$$= \int (\sec^7 x - 2\sec^5 x + \sec^3 x) \, dx$$

$$= \int \sec^7 x \, dx - 2 \int \sec^5 x \, dx + \int \sec^3 x \, dx$$

$$= \text{etc...}$$